Multiphysics Simulation Methods in Computer Graphics

Daniel Holz[1,2], Stefan Rhys Jeske[3], Fabian Löschner[3], Jan Bender[3], Yin Yang[4], Sheldon Andrews[2]

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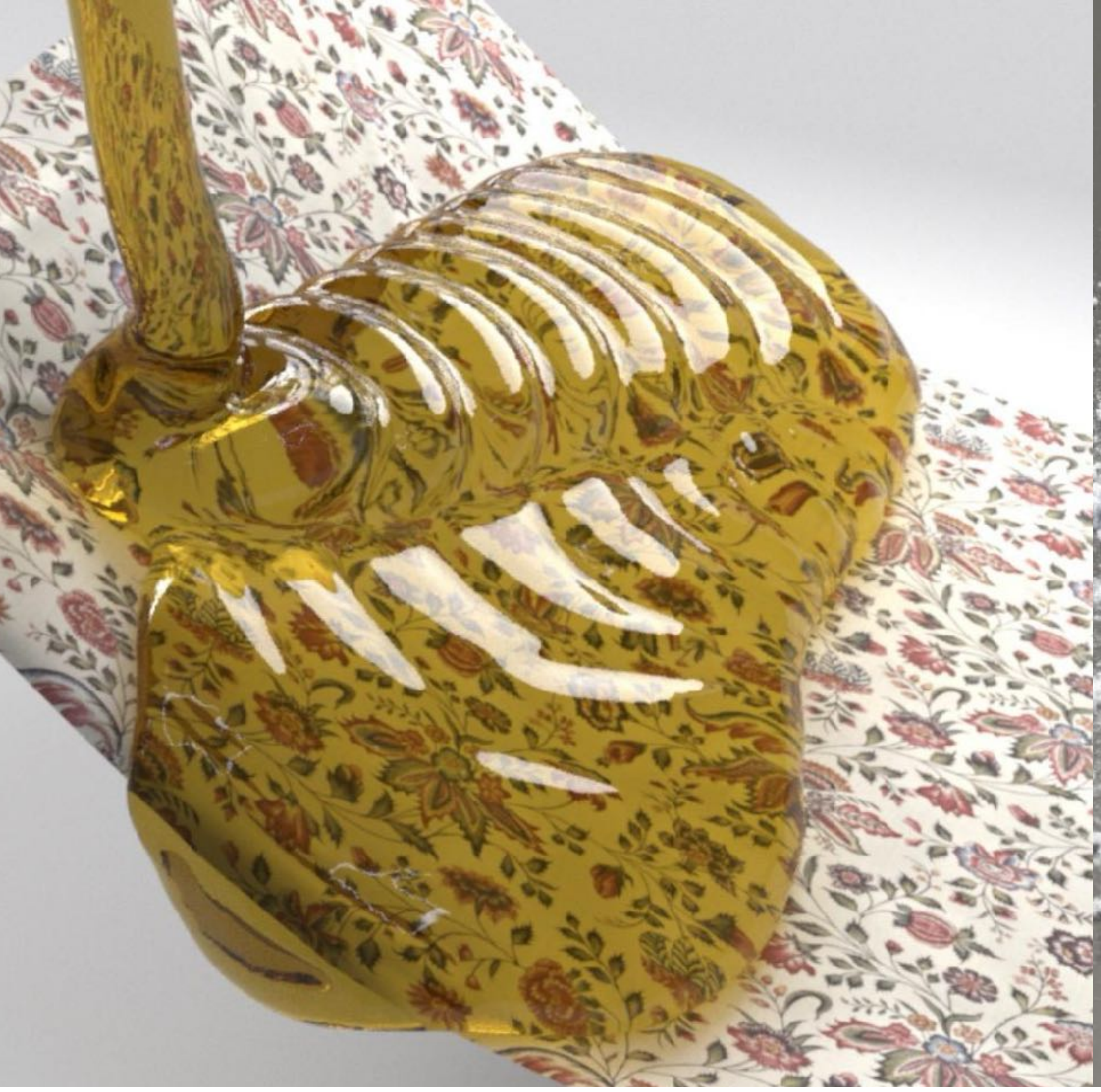


Part II
Energy-based multiphysics modeling

Daniel Holz[1,2], Stefan Rhys Jeske[3], **Fabian Löschner**[3], Jan Bender[3], Yin Yang[4], Sheldon Andrews[2]

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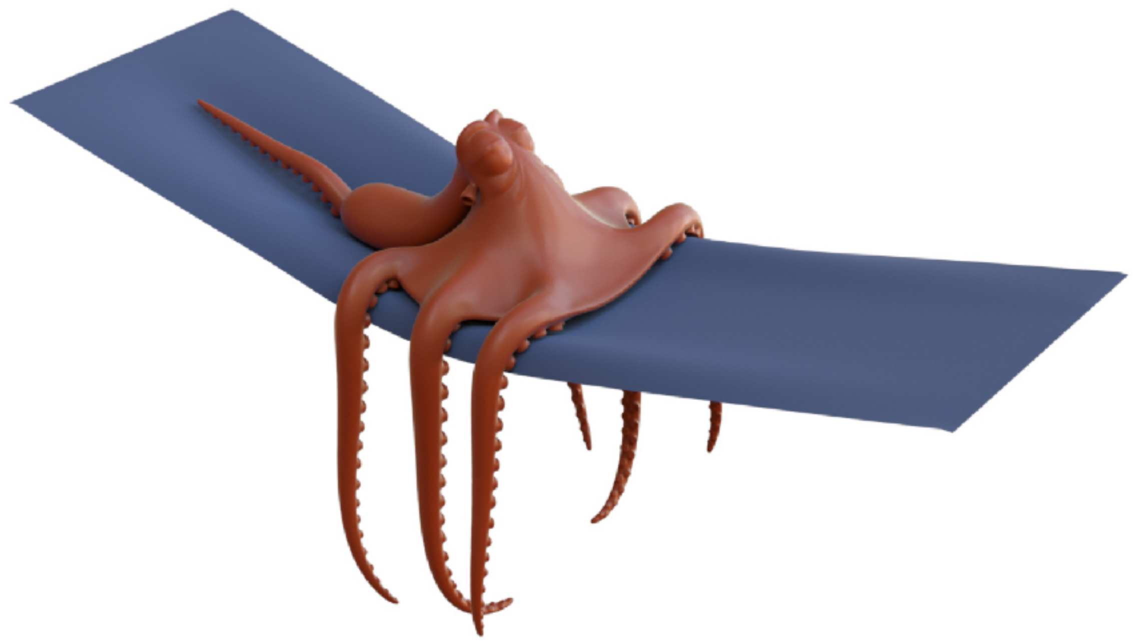
Elastic Energies in Computer Graphics

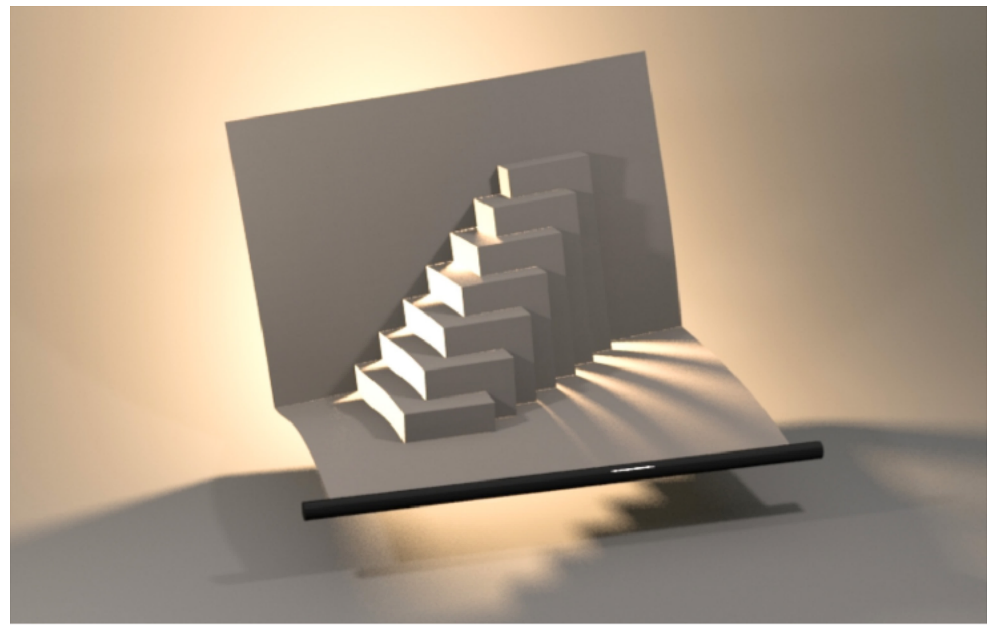
Deformable Solids



Dynamic Deformables [Kim, Eberle 2022]

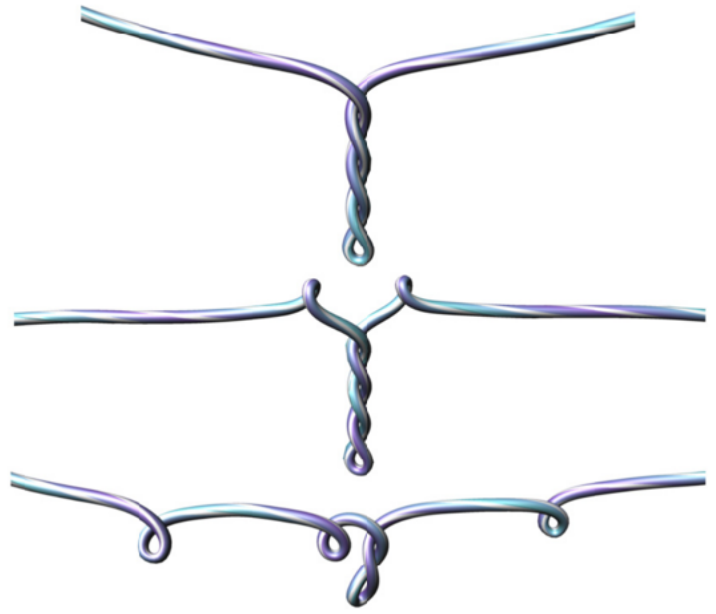
Cloth & Shells

Progressive Dynamics for Cloth and Shell Animation
[Zhang et al. 2024]



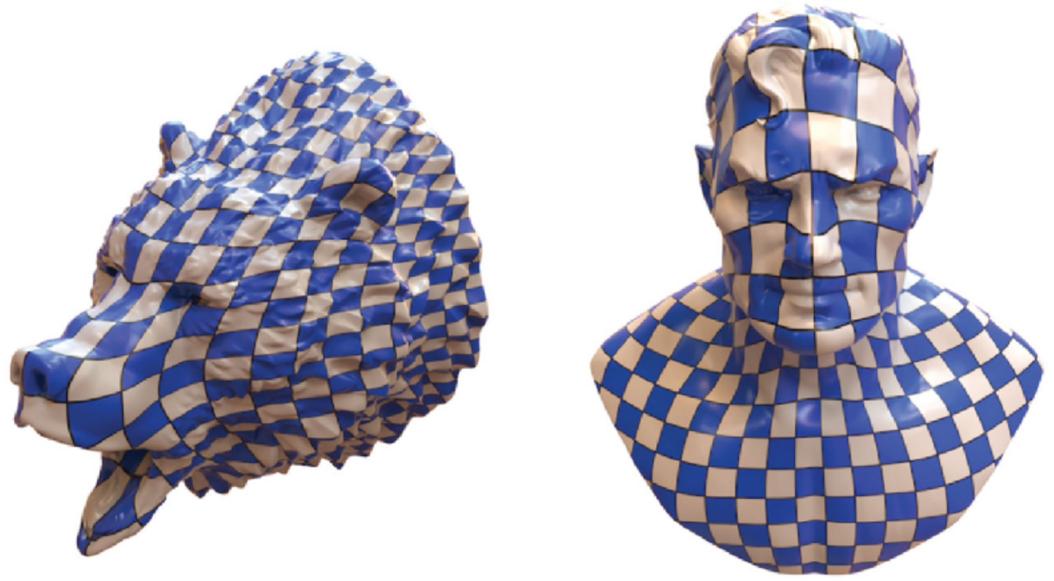
Second-Order Finite Elements for Deformable Surfaces
[Le et al. 2023]

Rods



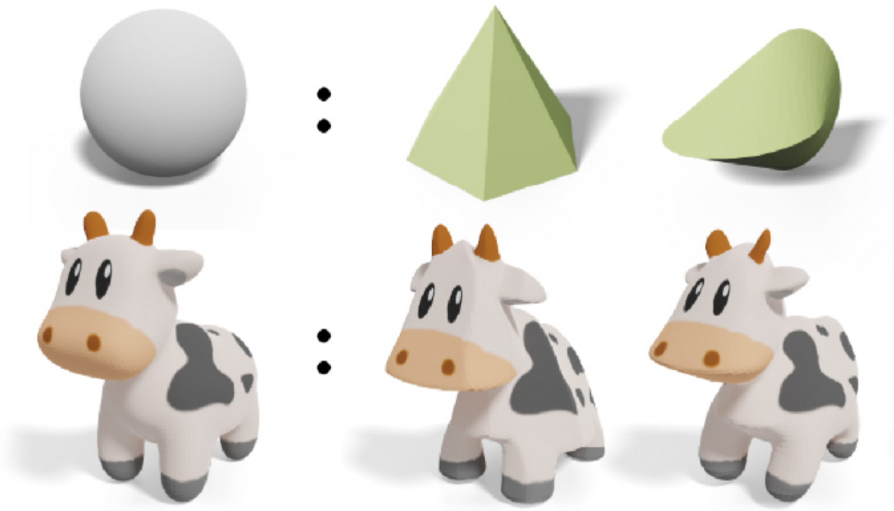
Discrete Elastic Rods
[Bergou et al. 2008]

Surface Parametrization



Analytic Eigensystems [Smith et al. 2019]

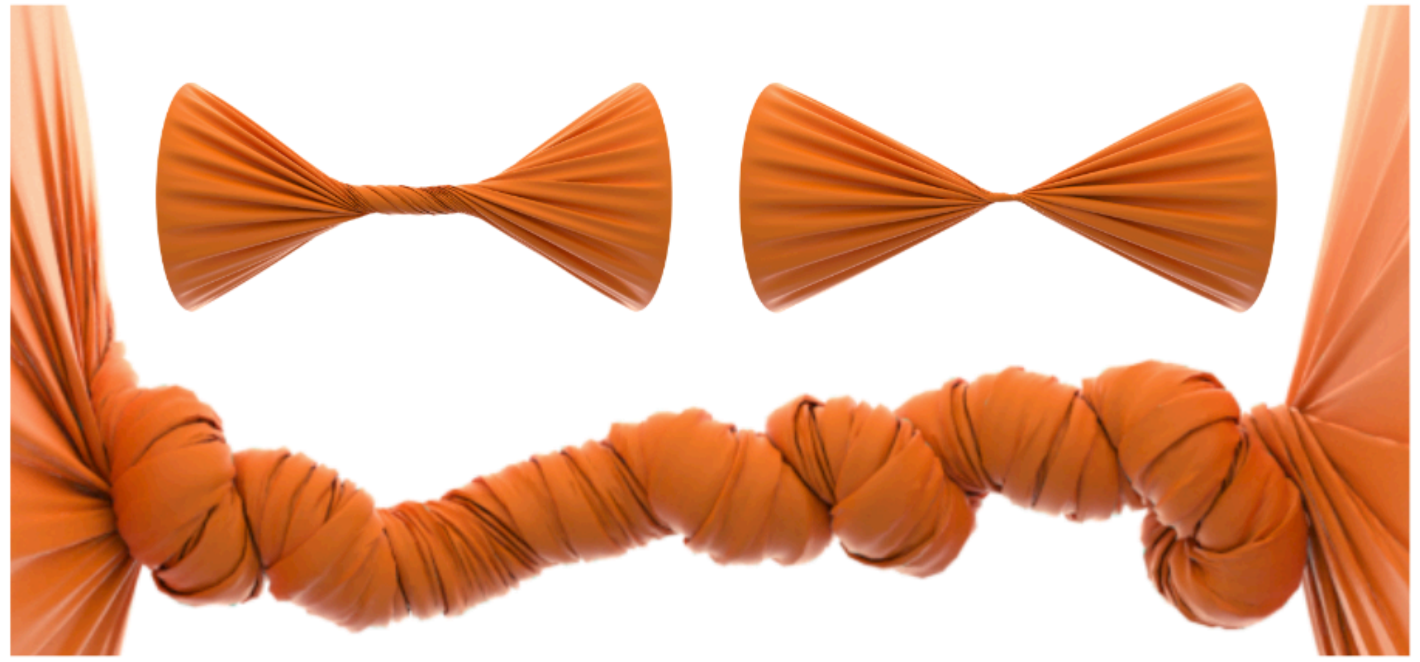
Shape Stylization & Optimization



Normal-Driven Spherical Shape Analogies
[Liu, Jacobson 2021]

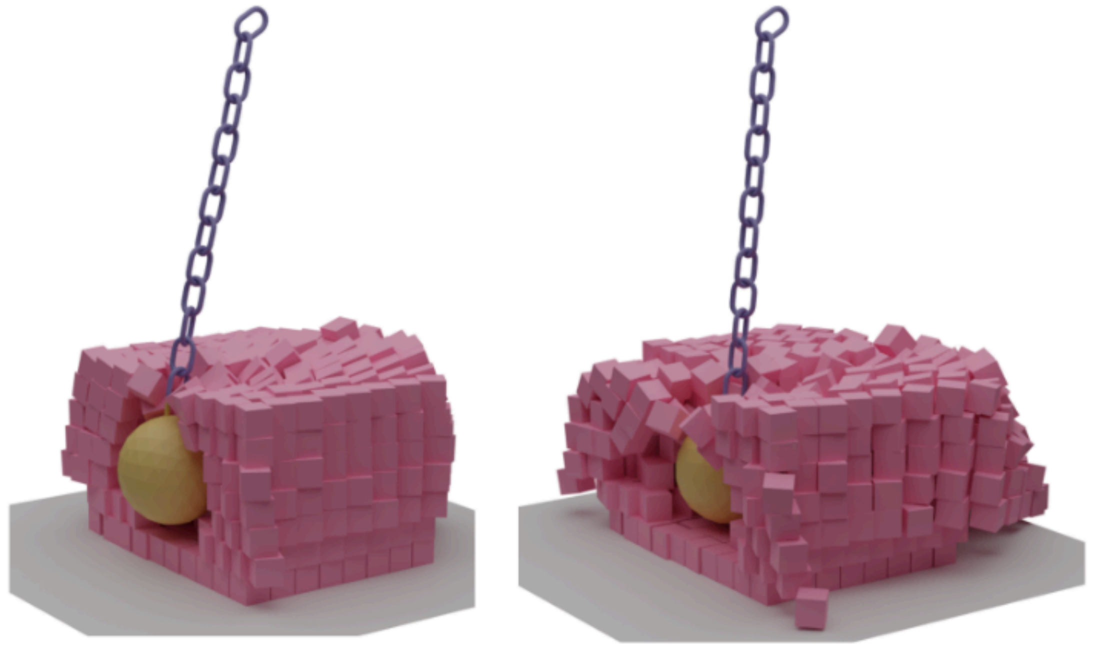
Multiphysics Energies

Contact



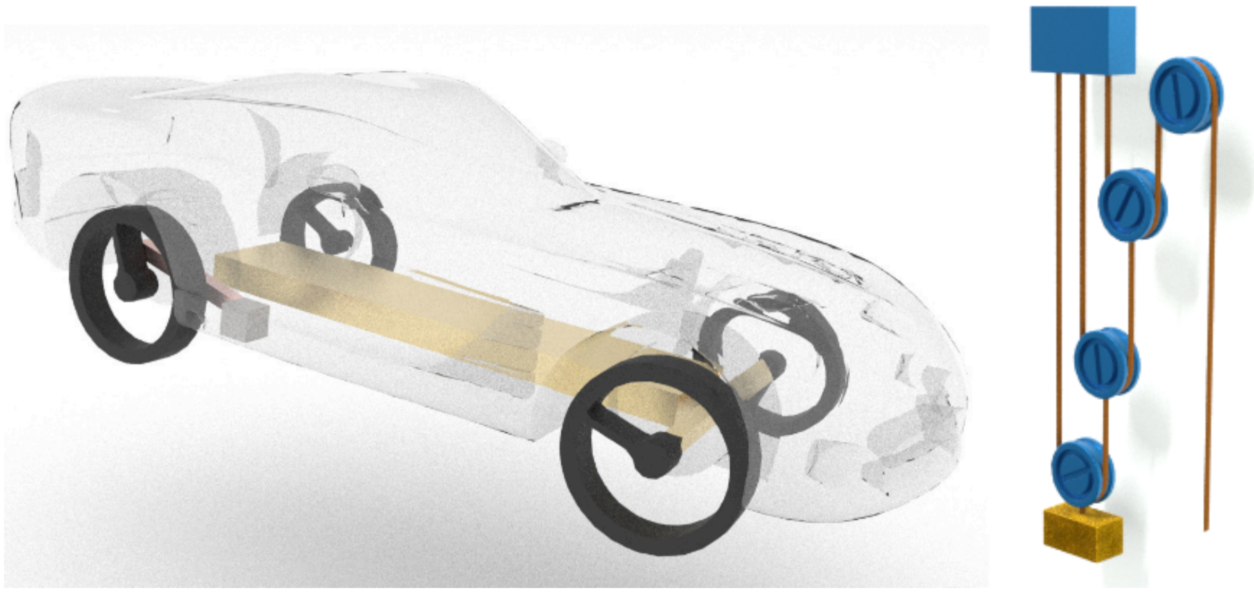
Codimensional Incremental Potential Contact [Li et al. 2021]

Rigid Bodies



Intersection-free Rigid Body Dynamics [Ferguson et al. 2021]

Multibody Systems



A Unified Newton Barrier Method for Multibody Dynamics [Chen et al. 2022]

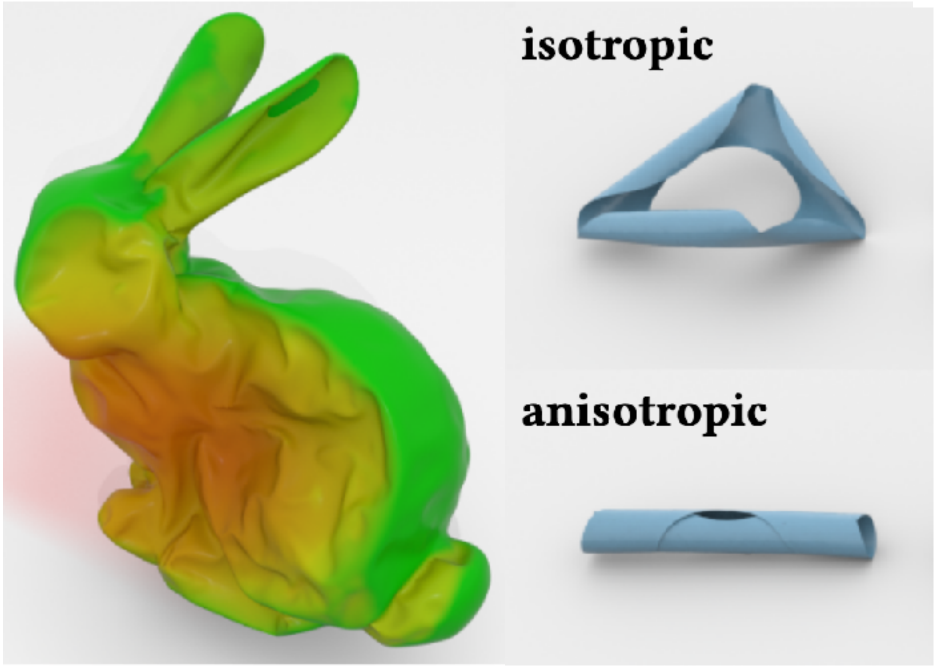
Fluids

A Contact Proxy Splitting Method for Lagrangian Solid-Fluid Coupling [Xie et al. 2023]

Magnetism

Strongly Coupled Simulation of Magnetic Rigid Bodies [Westhofen et al. 2024]

Heating & Wetting

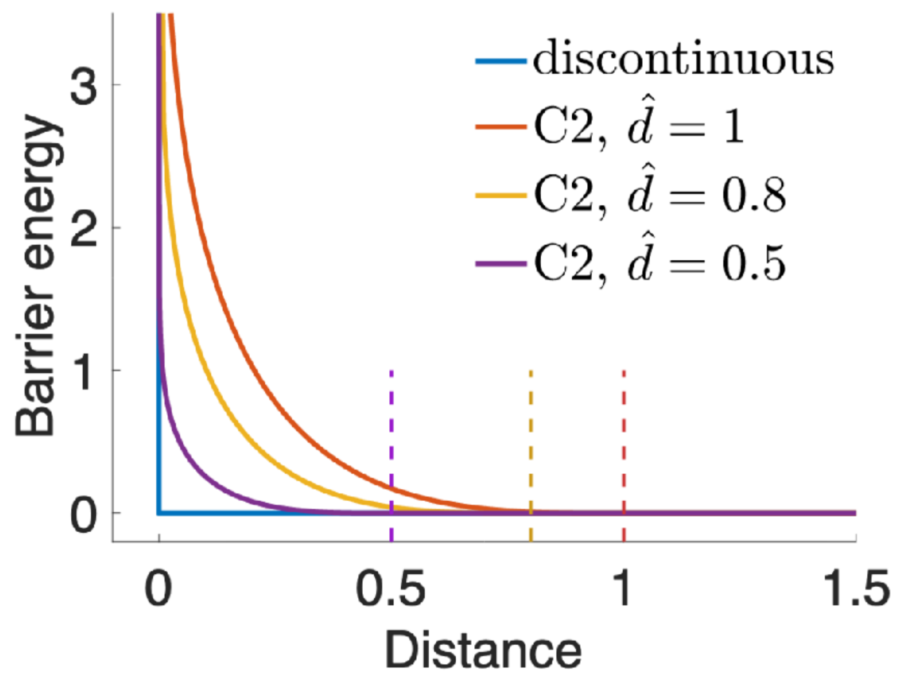


Physical Simulation of Environmentally Induced Thin Shell Deformation [Chen et al. 2018]

Energy-Based Multiphysics Modeling

Unifying mathematical formulation

- Physical systems modeled by **scalar potentials**
- Constraints modeled by **barrier potentials** or penalties

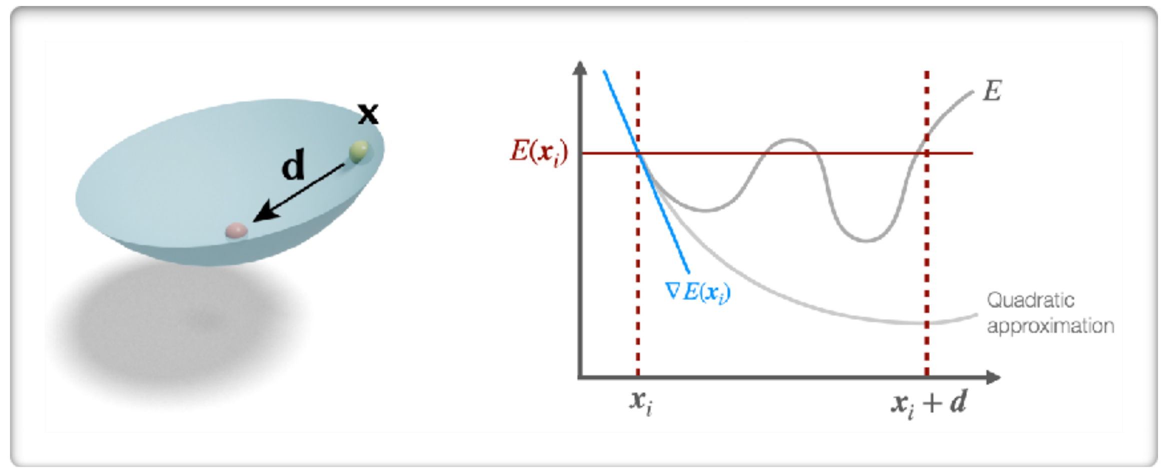


Incremental Potential Contact [Li et al. 2020]

Typically

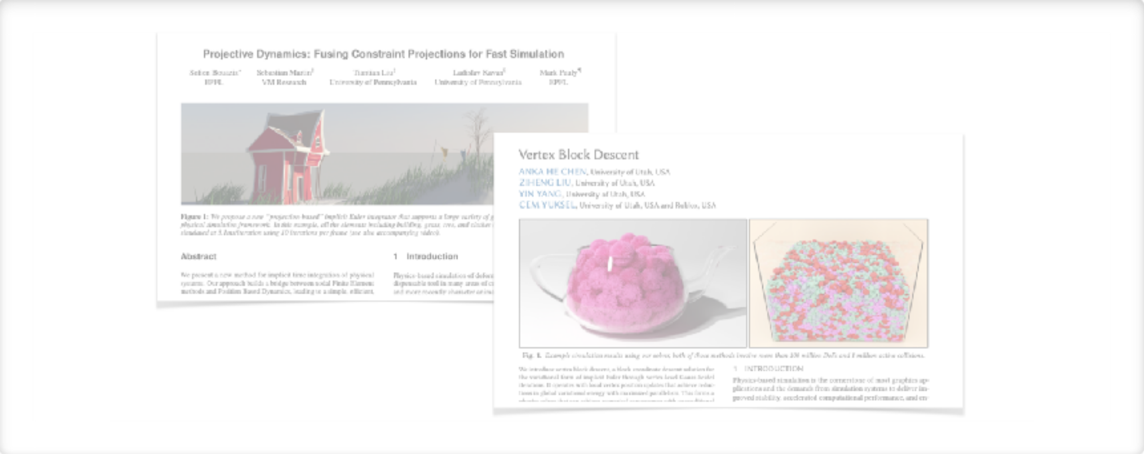
- …leads to fully implicit, **primal** formulation
- …solved as **unconstrained optimization** problem

Outline

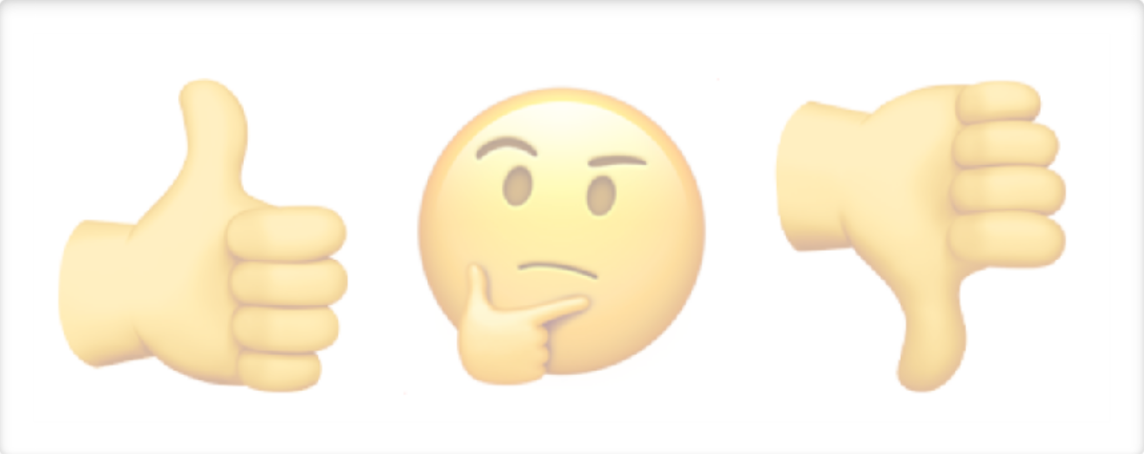


1. Mathematical foundations

2. Physical models and coupling



3. Related methods (VBD, PD)



4. Summary: Models & Properties

Optimization-Based Time Integration

- Example: Discretize Equations of Motion with BE

$$\mathbf{M}\ddot{\boldsymbol{x}} = \sum_i \boldsymbol{f}_i(\boldsymbol{x}) \quad \text{discretized as} \quad \mathbf{M}\frac{\boldsymbol{x} - \boldsymbol{x}^{\text{prev}} - \Delta t \boldsymbol{v}^{\text{prev}}}{\Delta t^2} - \sum_i \boldsymbol{f}_i(\boldsymbol{x}) = 0$$

- Define $E(\boldsymbol{x})$ such that $\nabla E(\boldsymbol{x}) = 0$ solves EoM

$$E(\boldsymbol{x}) = \frac{1}{2\Delta t^2}(\boldsymbol{x} - \tilde{\boldsymbol{x}})^{\mathsf{T}}\mathbf{M}(\boldsymbol{x} - \tilde{\boldsymbol{x}}) + \sum_i \phi_i(\boldsymbol{x}) \quad \text{where} \quad \boldsymbol{f}_i(\boldsymbol{x}) = -\nabla \phi_i(\boldsymbol{x})$$

Optimization-Based Time Integration

- **Incremental potential** of BE:

$$E(\boldsymbol{x}) = \frac{1}{2\Delta t^2}(\boldsymbol{x} - \tilde{\boldsymbol{x}})^\top \mathbf{M}(\boldsymbol{x} - \tilde{\boldsymbol{x}}) + \sum_i \phi_i(\boldsymbol{x})$$

- Minimize to solves balance of forces

$$\min_{\boldsymbol{x}} E(\boldsymbol{x})$$

- Physical effects modeled by scalar potentials $\phi_i(\boldsymbol{x})$ with

$$\boldsymbol{f}_i(\boldsymbol{x}) = -\nabla \phi_i(\boldsymbol{x})$$

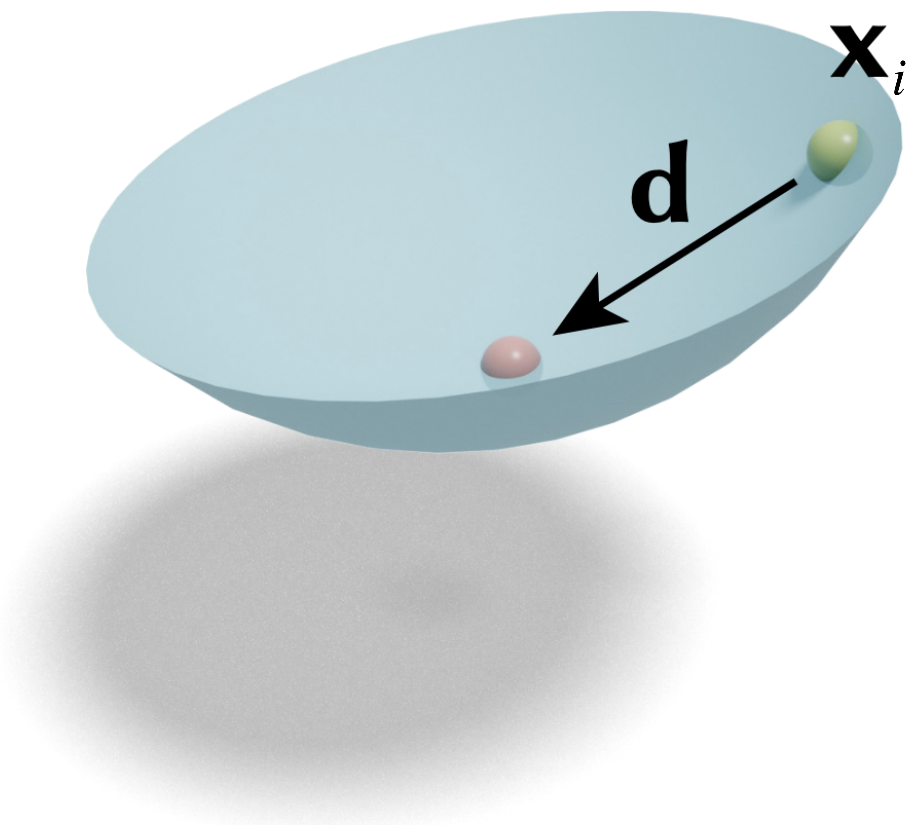
Minimizing the Incremental Potential

Newton's method: find direction $\boldsymbol{d}$ that minimizes quadratic approximation around $\boldsymbol{x}_i$

$$\min_{\boldsymbol{d}} E(\boldsymbol{x}_i) + \nabla E(\boldsymbol{x}_i)^\top \boldsymbol{d} + \frac{1}{2}\boldsymbol{d}^\top \mathbf{H}(\boldsymbol{x}_i)\,\boldsymbol{d} \quad \text{solved by} \quad \boldsymbol{d} = -\,\mathbf{H}(\boldsymbol{x}_i)^{-1}\,\nabla E(\boldsymbol{x}_i)$$

…if objective is convex

Problems in practice:



Introduction to Optimization for Simulation
Honglin Chen 2024

💥 **Forces are non-linear:** full step $\boldsymbol{d}$ might overshoot actual minimum

- 👉 Solution: Line search using $E(\boldsymbol{x})$

💥 **Indefiniteness:** Objective might not be convex everywhere

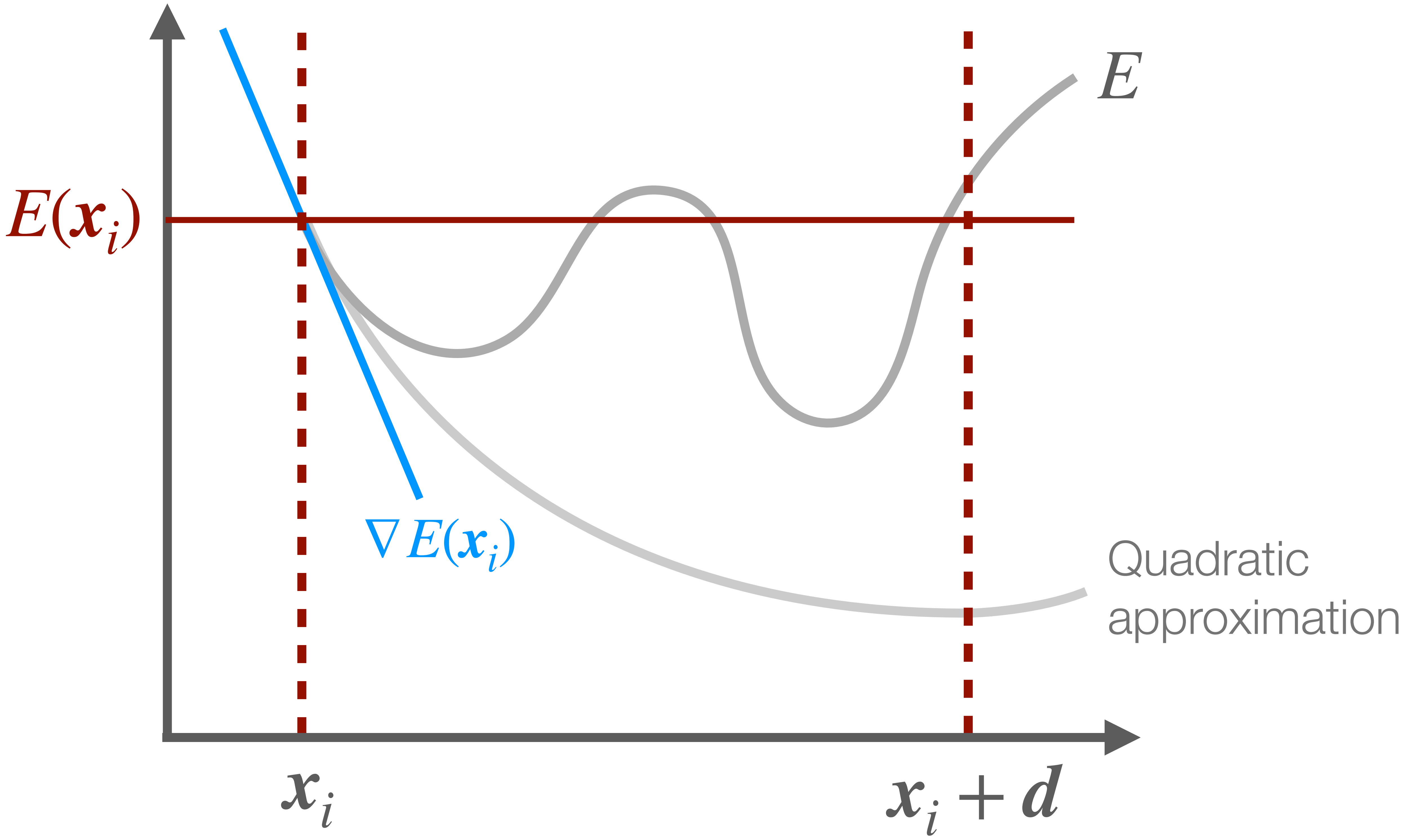
- 👉 Solution: "Projected Newton", project $\mathbf{H}(\boldsymbol{x})$ to positive semi-definiteness (PSD)

Backtracking Line Search

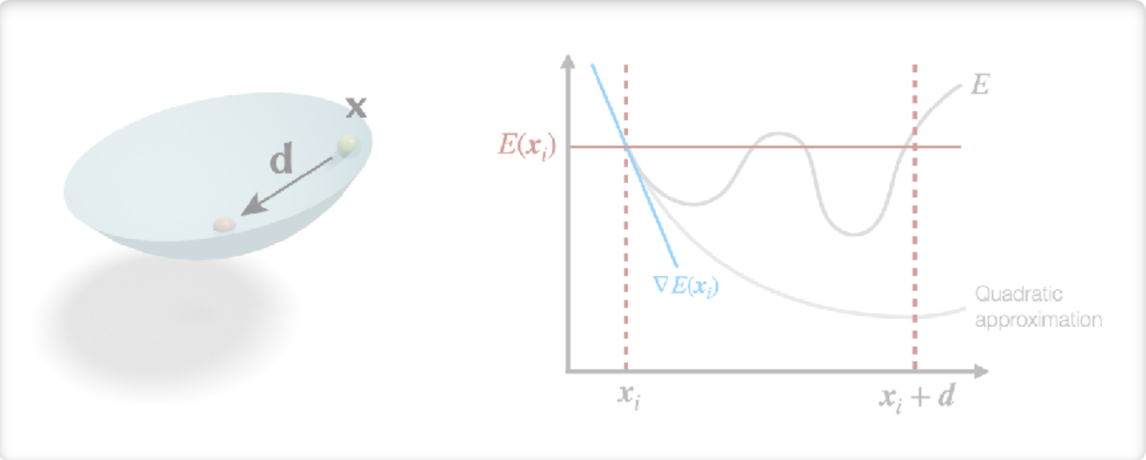
Goal: Find step length $\alpha \in [0,1]$ such that step $\alpha\,\boldsymbol{d}$ decreases objective

→ i.e. $E(\boldsymbol{x}_i + \alpha \boldsymbol{d}) < E(\boldsymbol{x}_i)$

- Halve α until condition satisfied
- Simplest form of Armijo condition
- CCD can be incorporated

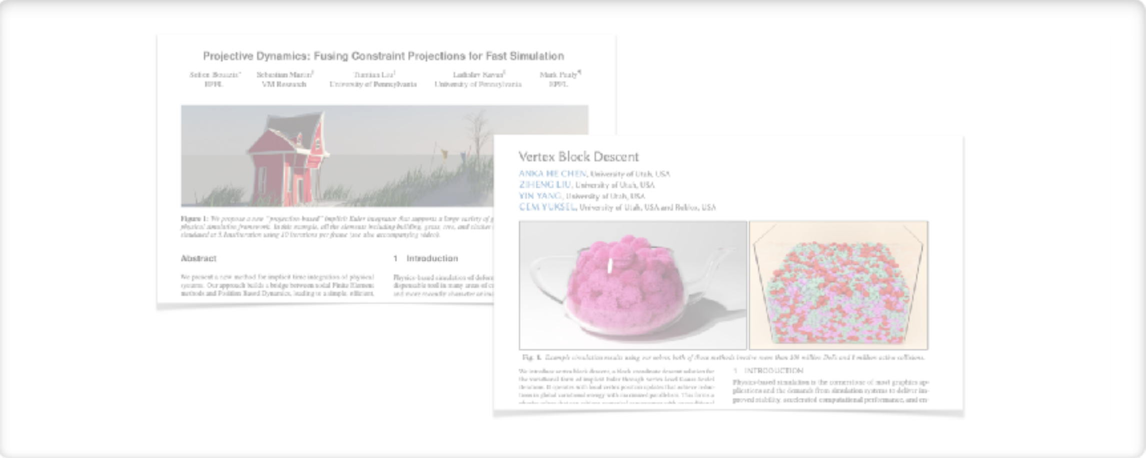


Outline



1. Mathematical foundations

2. Physical models and coupling



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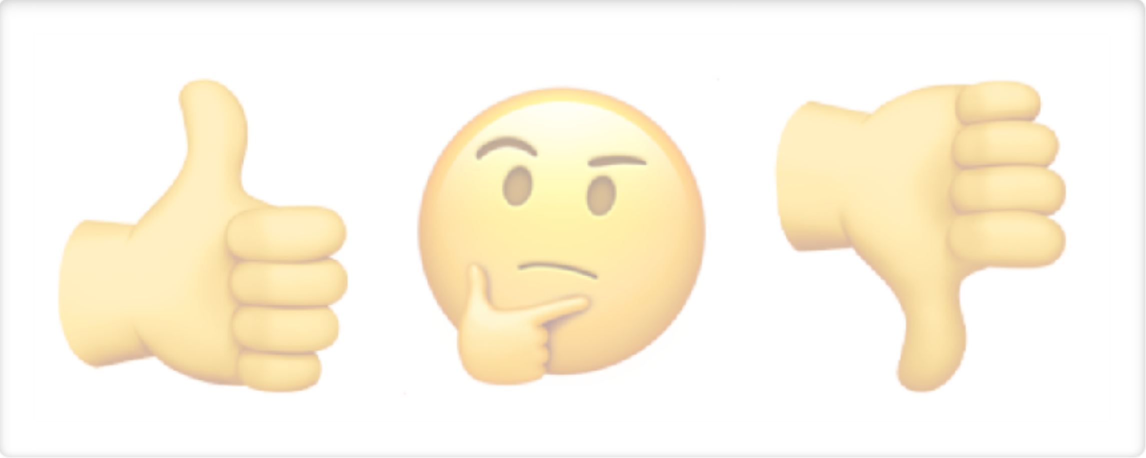
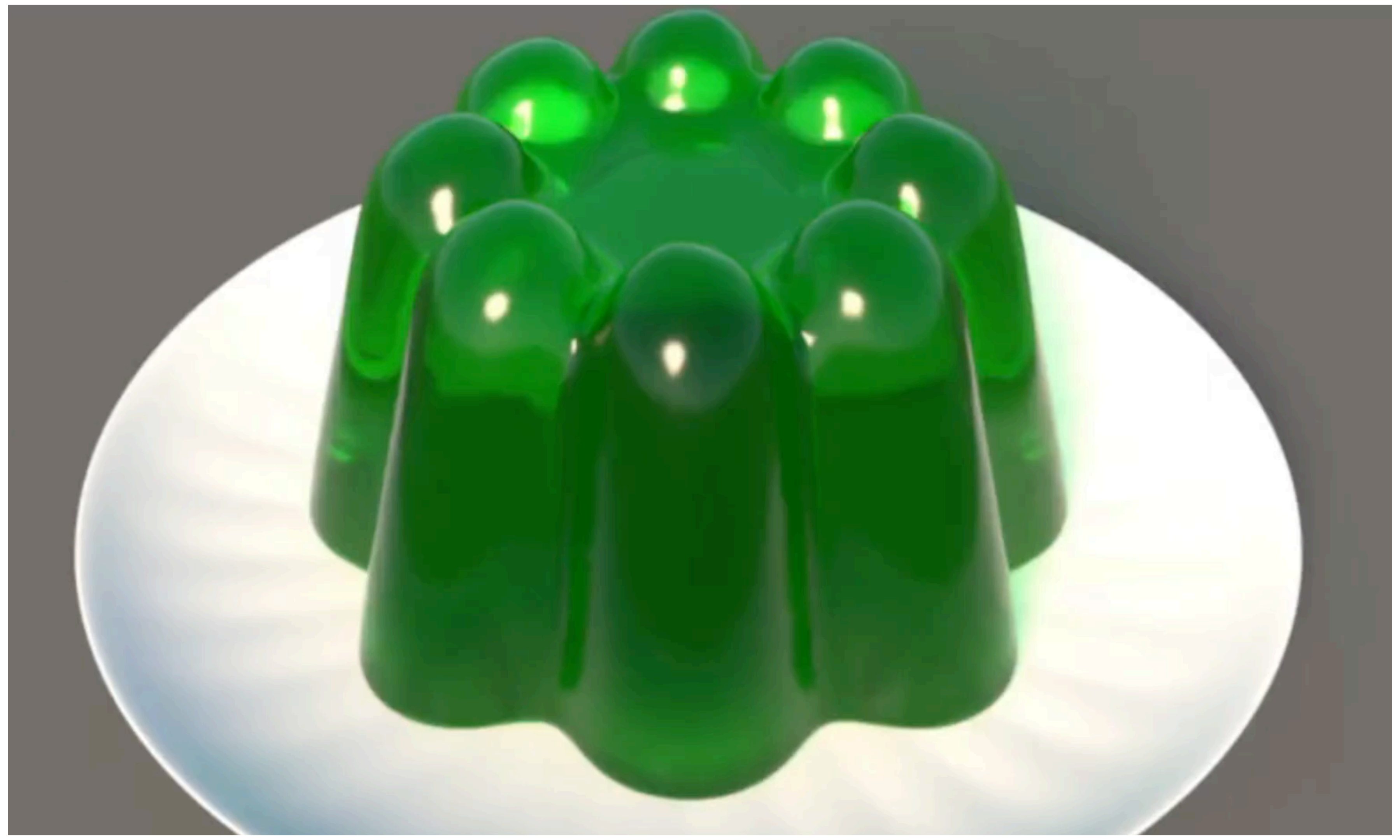


4. Summary: Models & Properties

Elastic Deformations

- Naturally described using energies
- Change in shape → change in internal energy
- Simple spring-based models
  - → e.g. $E_{\text{spring}} = \frac{1}{2}k\|\boldsymbol{x}_i - \boldsymbol{x}_j\|^2/l_0$

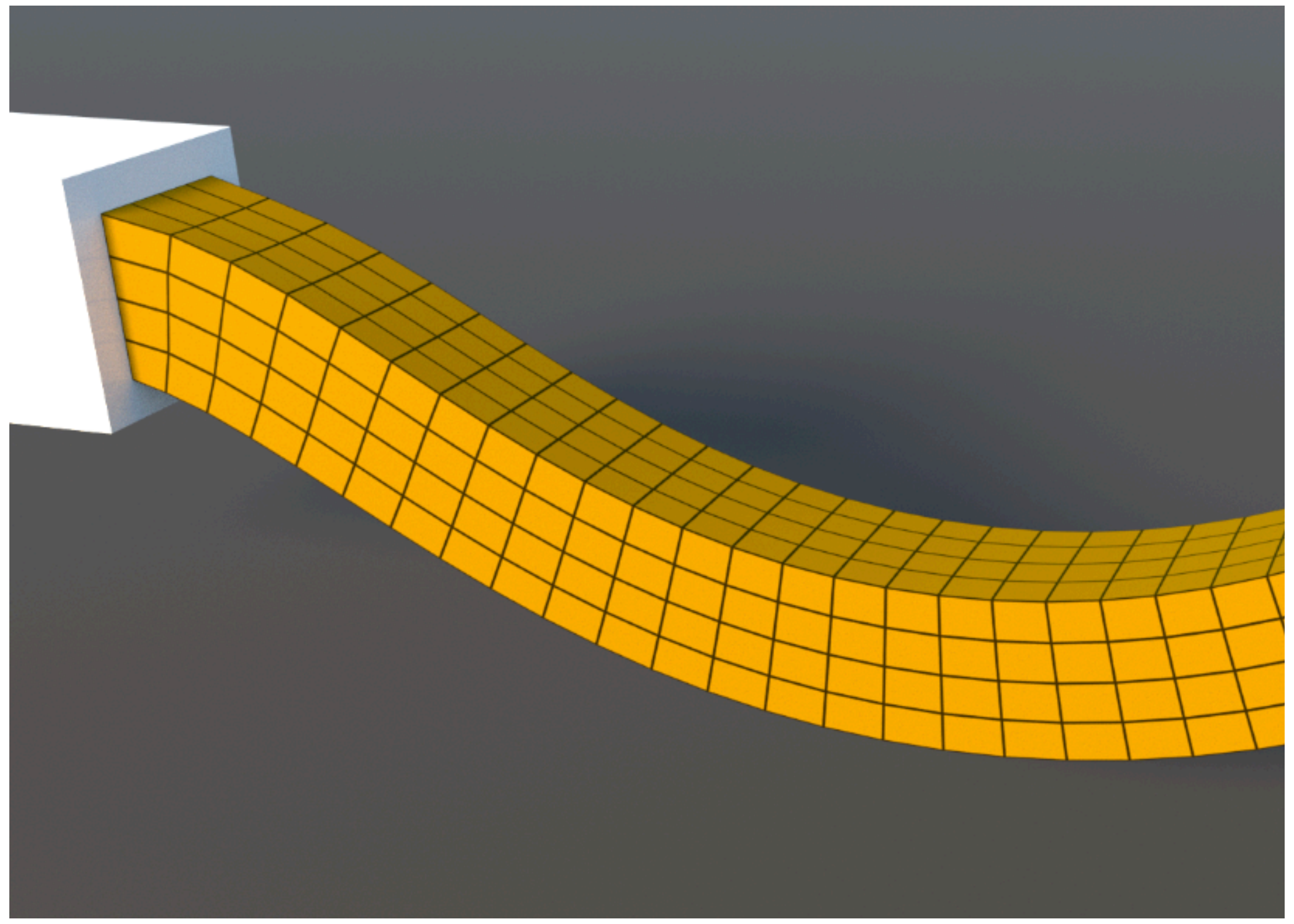


Higher-Order Time Integration for Deformable Solids [Löschner et al. 2020]

- Continuum-based models: **strain energy densities**
  - → e.g. Stable Neohookean: $\Psi_{\text{SNH}} = \frac{\mu}{2}(\text{tr}(\mathbf{F}^\top\mathbf{F}) - 3) - \mu(\det\mathbf{F} - 1) + \frac{\lambda}{2}(\det\mathbf{F} - 1)^2$

Elastic Deformables using FEM

- Continuum-based models: **strain energy densities**
  - → e.g. Stable Neohookean: $\Psi_{\text{SNH}} = \frac{\mu}{2}(\text{tr}(\mathbf{F}^\top \mathbf{F}) - 3) - \mu(\det \mathbf{F} - 1) + \frac{\lambda}{2}(\det \mathbf{F} - 1)^2$
- Discretization with FEM yields **energy per element**
  - → e.g. for linear elements: $E_{\text{strain}}(\boldsymbol{x}) = \sum_i^{N_{\text{Ele}}} V_i \Psi(\mathbf{F}_i(\boldsymbol{x}))$
- Forces: $\boldsymbol{f} = -\nabla E_{\text{strain}}$, stiffness matrix: $\mathbf{K} = -\frac{\partial^2 E_{\text{strain}}}{\partial \boldsymbol{x}^2}$
- Minimization solves balance of forces



Modelling dynamic contact

- Incremental Potential Contact [Li et al. 2020]

  ↳ Goal: intersection-free simulations in unconstrained optimization

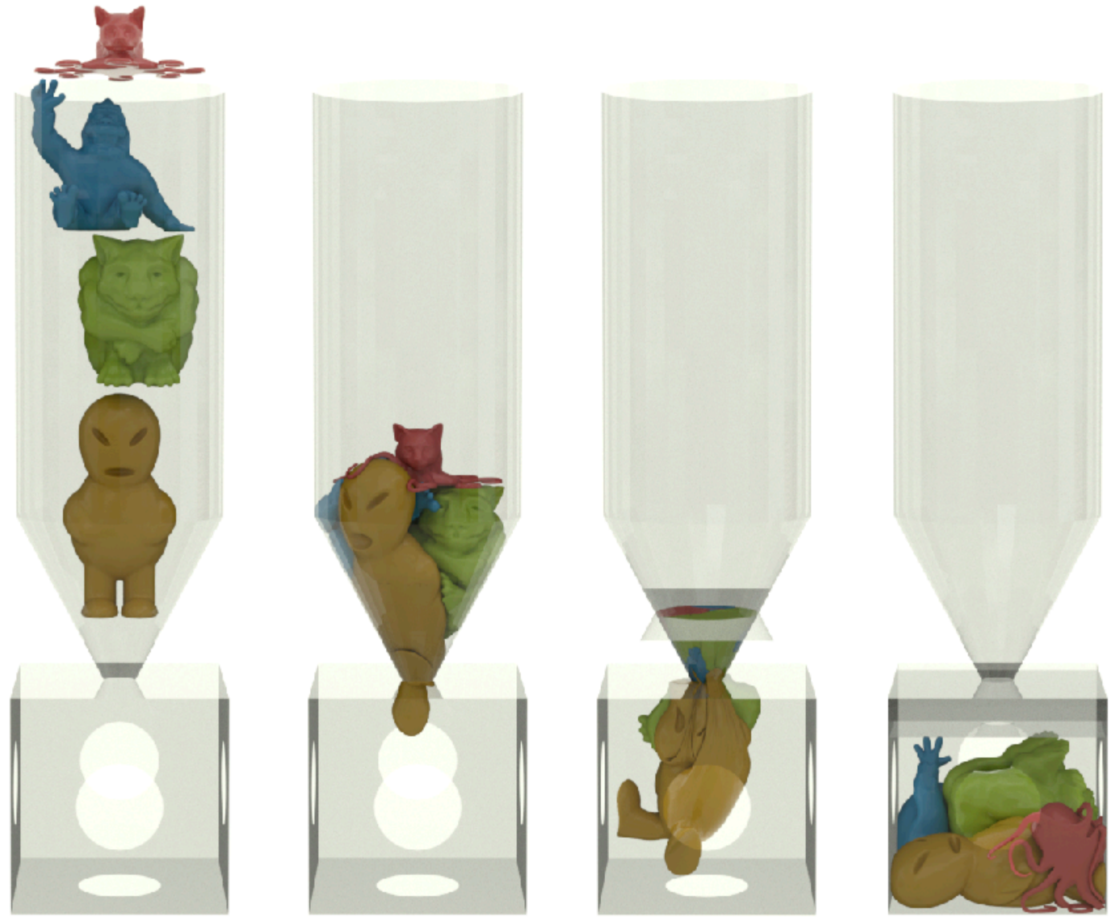
- Introduce smoothly clamped barriers:

$$b(d,\hat{d}) = \begin{cases} -(d-\hat{d})^2 \ln\left(\frac{d}{\hat{d}}\right), & 0 < d < \hat{d} \\ 0 & d \geq \hat{d} \end{cases}$$

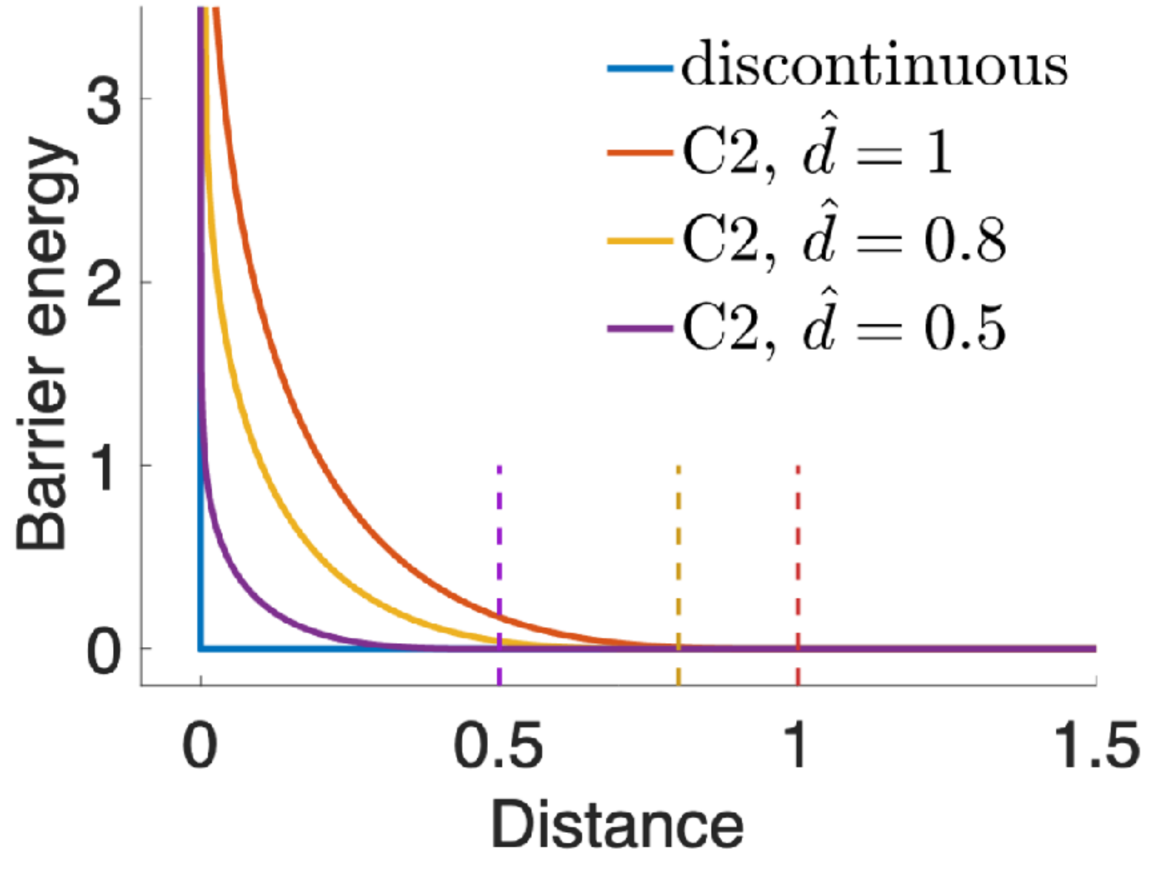
  With contact potential $E_C(\boldsymbol{x}) = \kappa \sum_{k \in C} b(d_k(\boldsymbol{x}))$

- Apply barriers to all triangle-vertex and edge-edge pairs $k \in C$

- Filtered line search: CCD determines upper bound for step

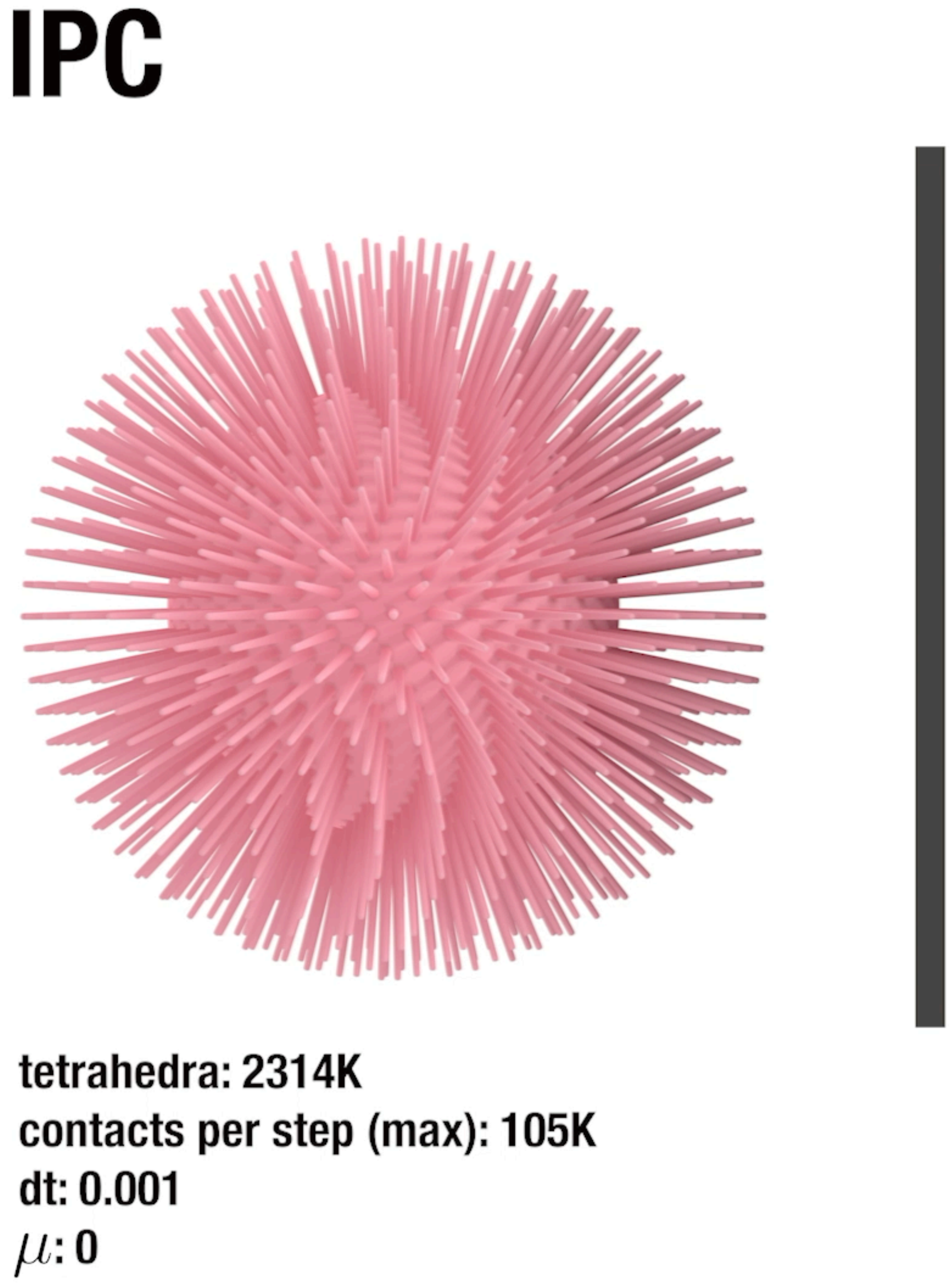


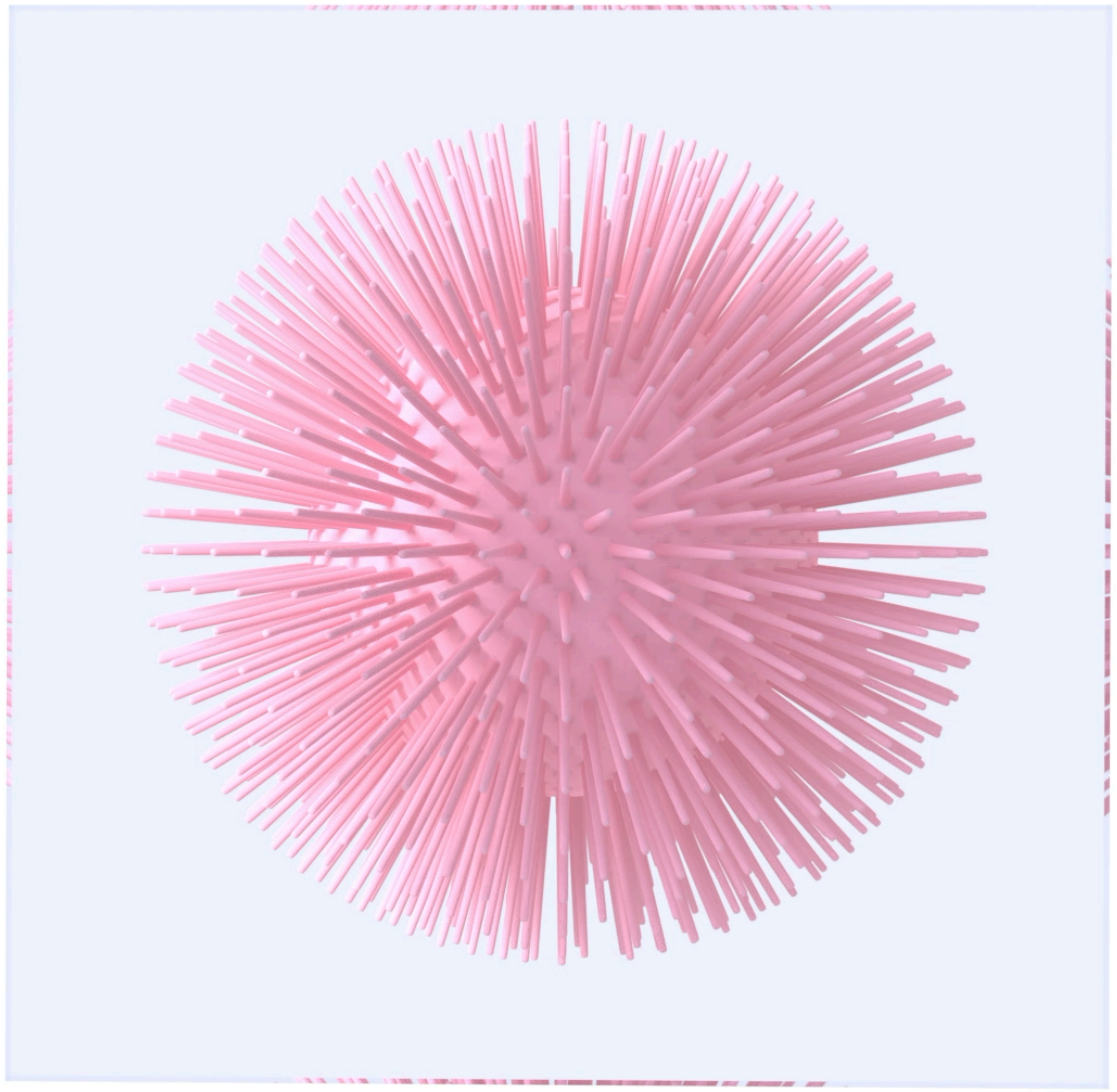
Incremental Potential Contact [Li et al. 2020]



Incremental Potential Contact [Li et al. 2020]

Intersection-free dynamic contact

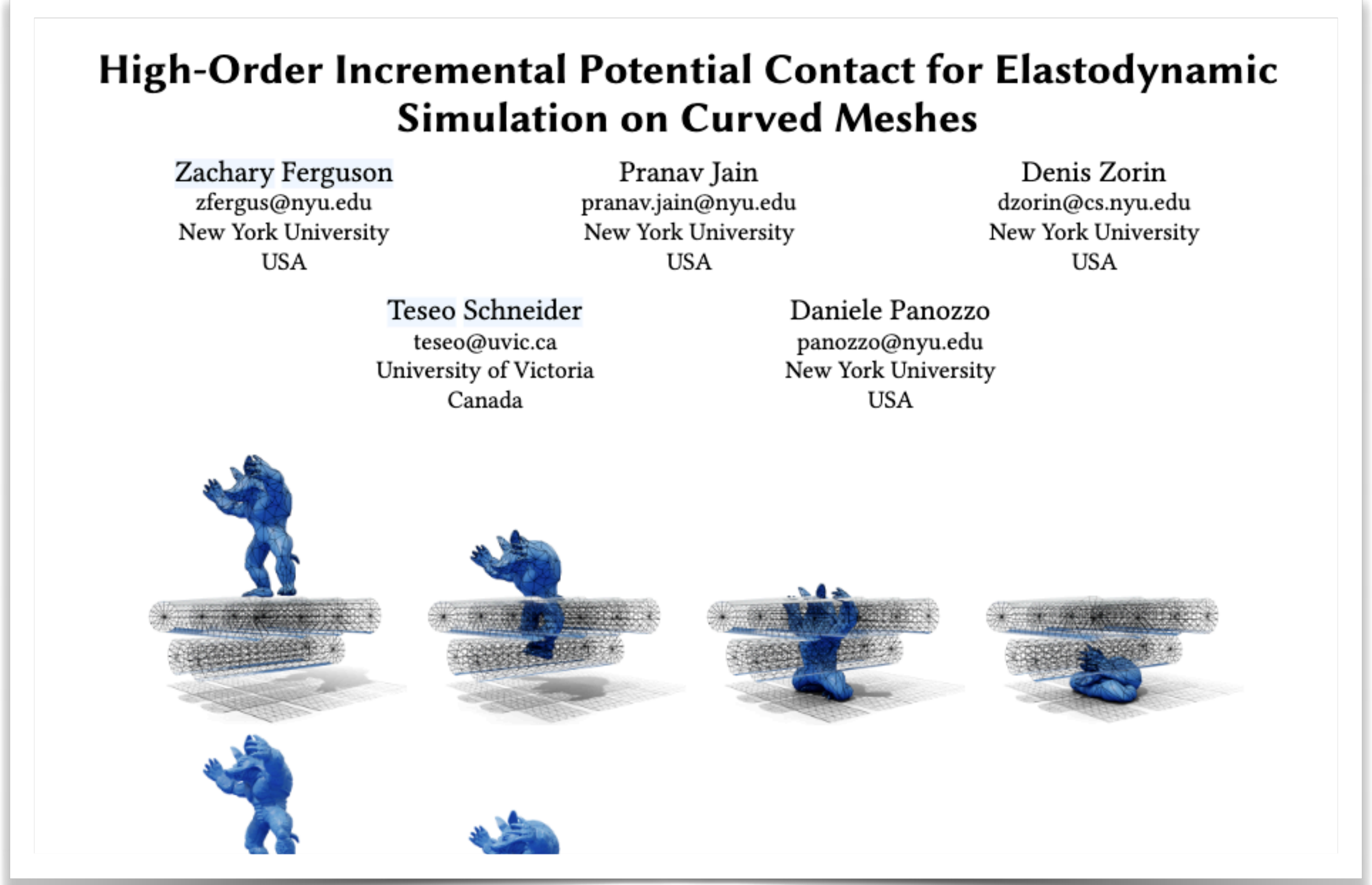




Intersection-free dynamic contact

tetrahedra: 181K
contacts per step (max): 277K
dt: 0.01
μ: 0

Intersection-free dynamic contact



High-Order Incremental Potential Contact for Elastodynamic Simulation on Curved Meshes

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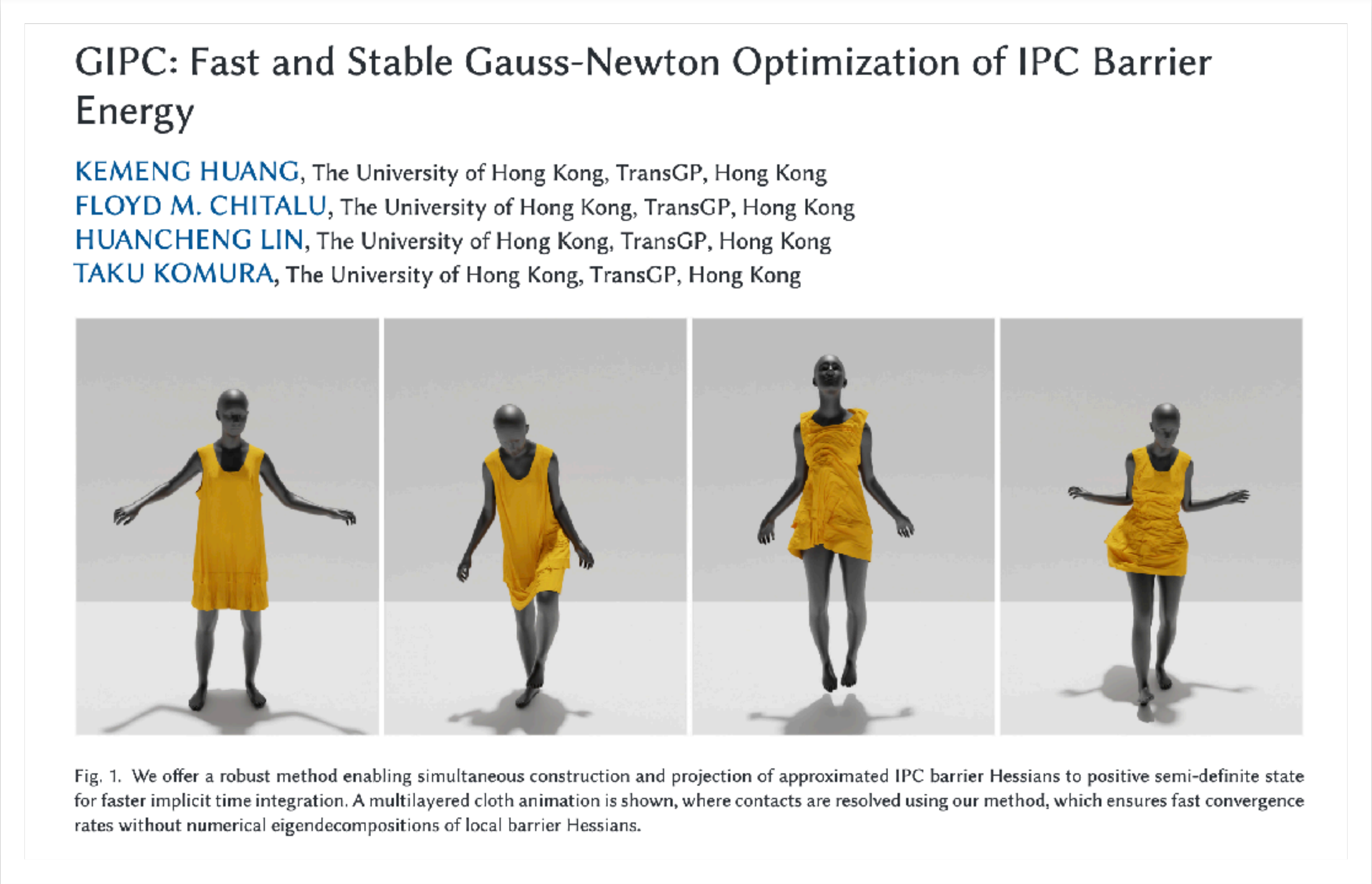

GIPC: Fast and Stable Gauss-Newton Optimization of IPC Barrier Energy

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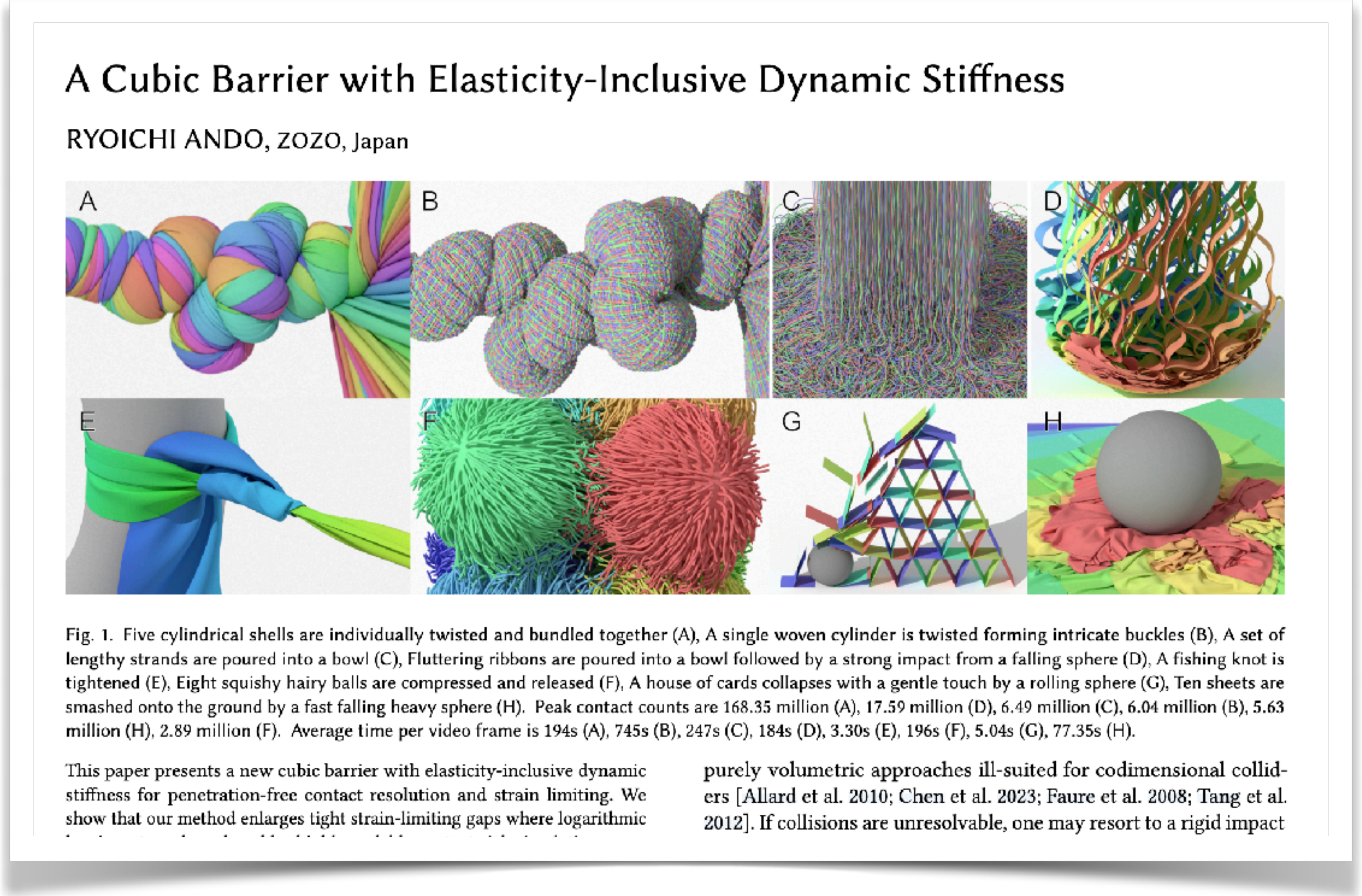
FLOYD M. CHITALU, The University of Hong Kong, TransGP, Hong Kong
HUANCHENG LIN, The University of Hong Kong, TransGP, Hong Kong
TAKU KOMURA, The University of Hong Kong, TransGP, Hong Kong

Fig. 1. We offer a robust method enabling simultaneous construction and projection of approximated IPC barrier Hessians to positive semi-definite state for faster implicit time integration. A multilayered cloth animation is shown, where contacts are resolved using our method, which ensures fast convergence rates without numerical eigendecompositions of local barrier Hessians.



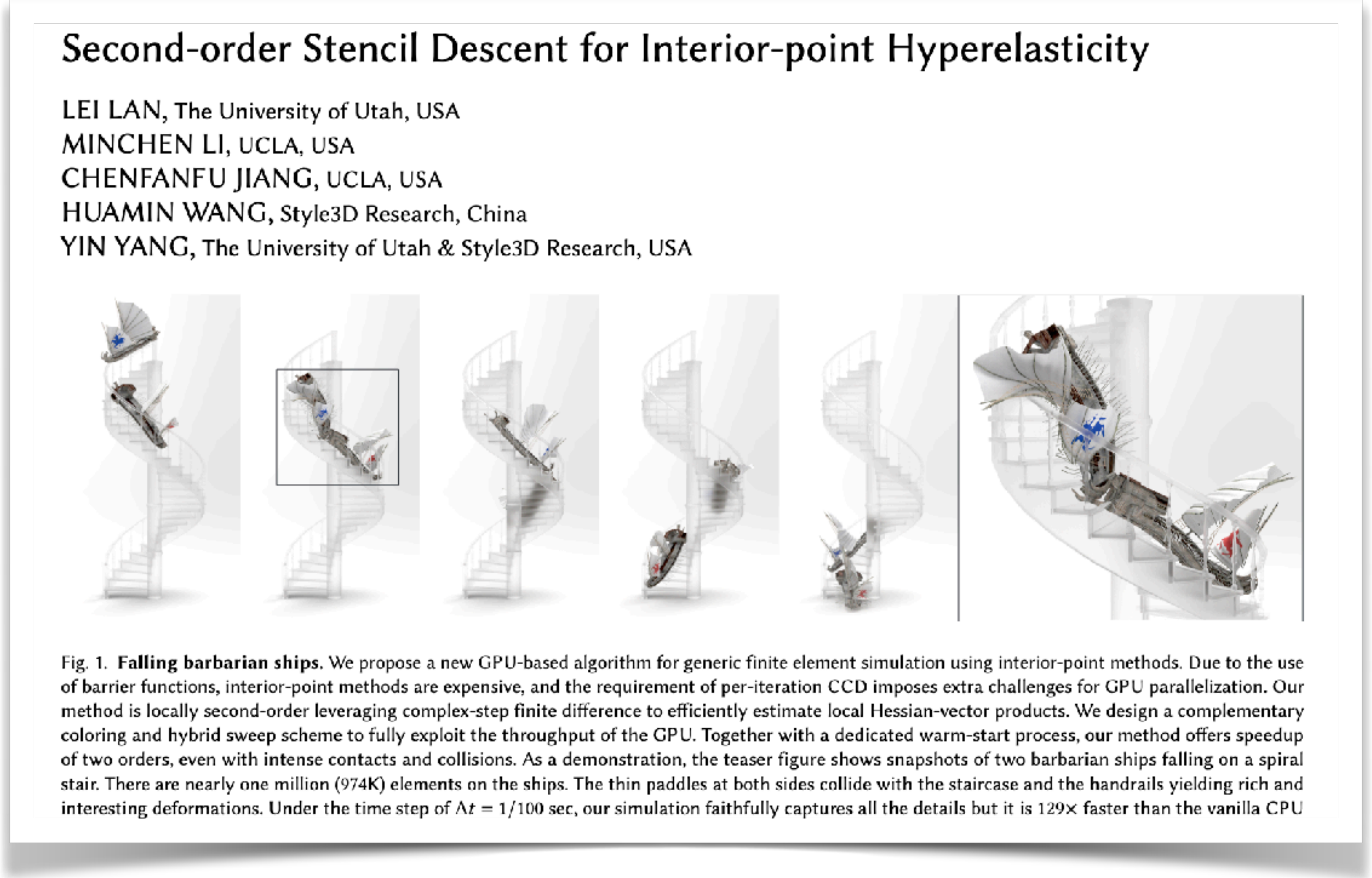
A Cubic Barrier with Elasticity-Inclusive Dynamic Stiffness

RYOICHI ANDO, ZOZO, Japan

Fig. 1. Five cylindrical shells are individually twisted and bundled together (A). A single woven cylinder is twisted forming intricate buckles (B). A set of lengthy strands are poured into a bowl (C). Fluttering ribbons are poured into a bowl followed by a strong impact from a falling sphere (D). A fishing knot is tightened (E). Eight squishy hairy balls are compressed and released (F). A house of cards collapses with a gentle touch by a rolling sphere (G). Ten sheets are smashed onto the ground by a fast falling heavy sphere (H). Peak contact counts are 168.35 million (A), 17.59 million (D), 6.49 million (C), 6.04 million (B), 5.63 million (H), 2.89 million (F). Average time per video frame is 194s (A), 745s (B), 247s (C), 184s (D), 3.30s (E), 196s (F), 5.04s (G), 77.35s (H).

This paper presents a new cubic barrier with elasticity-inclusive dynamic stiffness for penetration-free contact resolution and strain limiting. We show that our method enlarges tight strain-limiting gaps where logarithmic

purely volumetric approaches ill-suited for codimensional colliders [Allard et al. 2010; Chen et al. 2023; Faure et al. 2008; Tang et al. 2012]. If collisions are unresolvable, one may resort to a rigid impact



Second-order Stencil Descent for Interior-point Hyperelasticity

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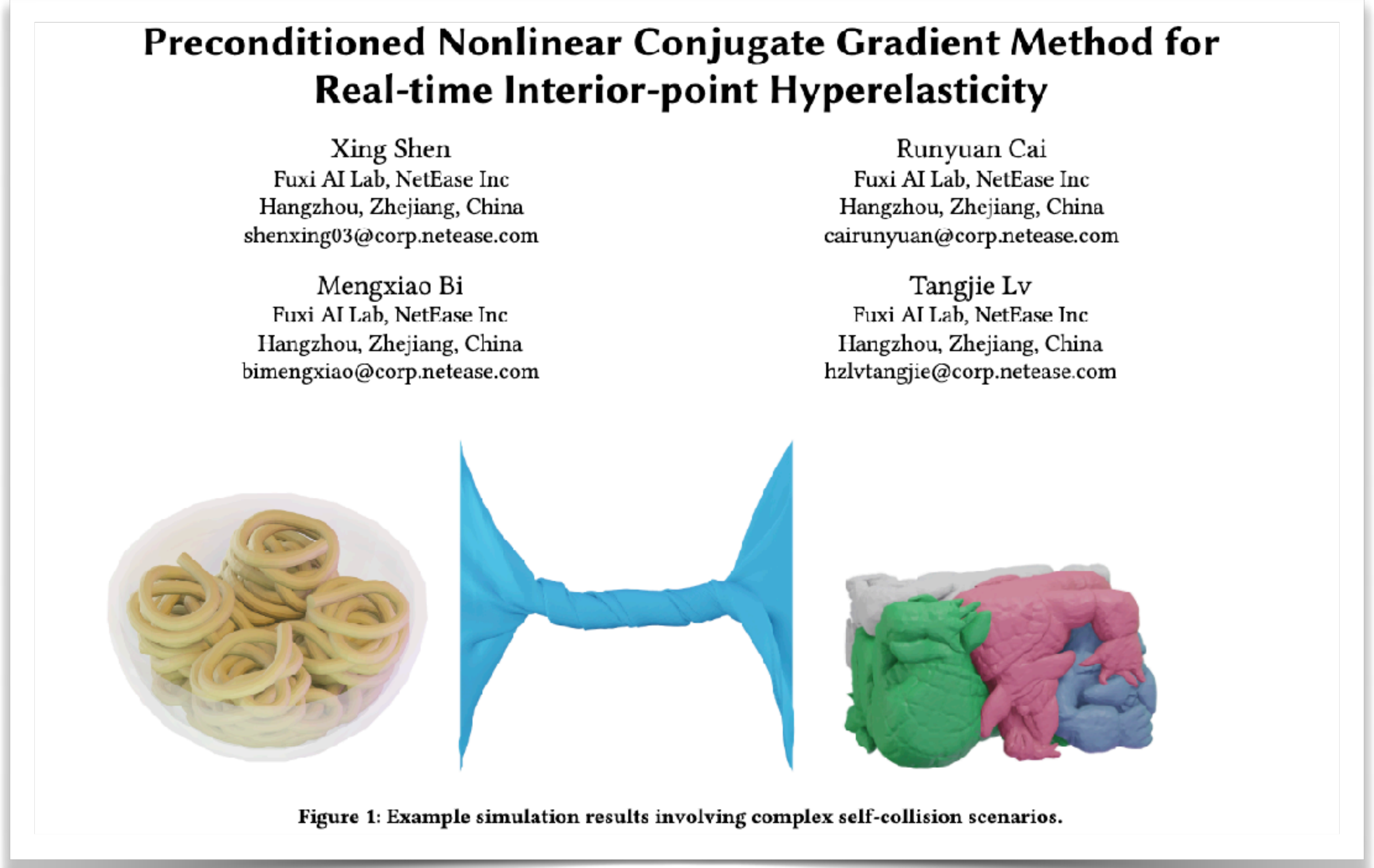
CHENFANFU JIANG, UCLA, USA
HUAMIN WANG, Style3D Research, China
YIN YANG, The University of Utah & Style3D Research, USA

Fig. 1. **Falling barbarian ships.** We propose a new GPU-based algorithm for generic finite element simulation using interior-point methods. Due to the use of barrier functions, interior-point methods are expensive, and the requirement of per-iteration CCD imposes extra challenges for GPU parallelization. Our method is locally second-order leveraging complex-step finite difference to efficiently estimate local Hessian-vector products. We design a complementary coloring and hybrid sweep scheme to fully exploit the throughput of the GPU. Together with a dedicated warm-start process, our method offers speedup of two orders, even with intense contacts and collisions. As a demonstration, the teaser figure shows snapshots of two barbarian ships falling on a spiral stair. There are nearly one million (974K) elements on the ships. The thin paddles at both sides collide with the staircase and the handrails yielding rich and interesting deformations. Under the time step of $\Delta t = 1/100$ sec, our simulation faithfully captures all the details but it is 129× faster than the vanilla CPU



Preconditioned Nonlinear Conjugate Gradient Method for Real-time Interior-point Hyperelasticity

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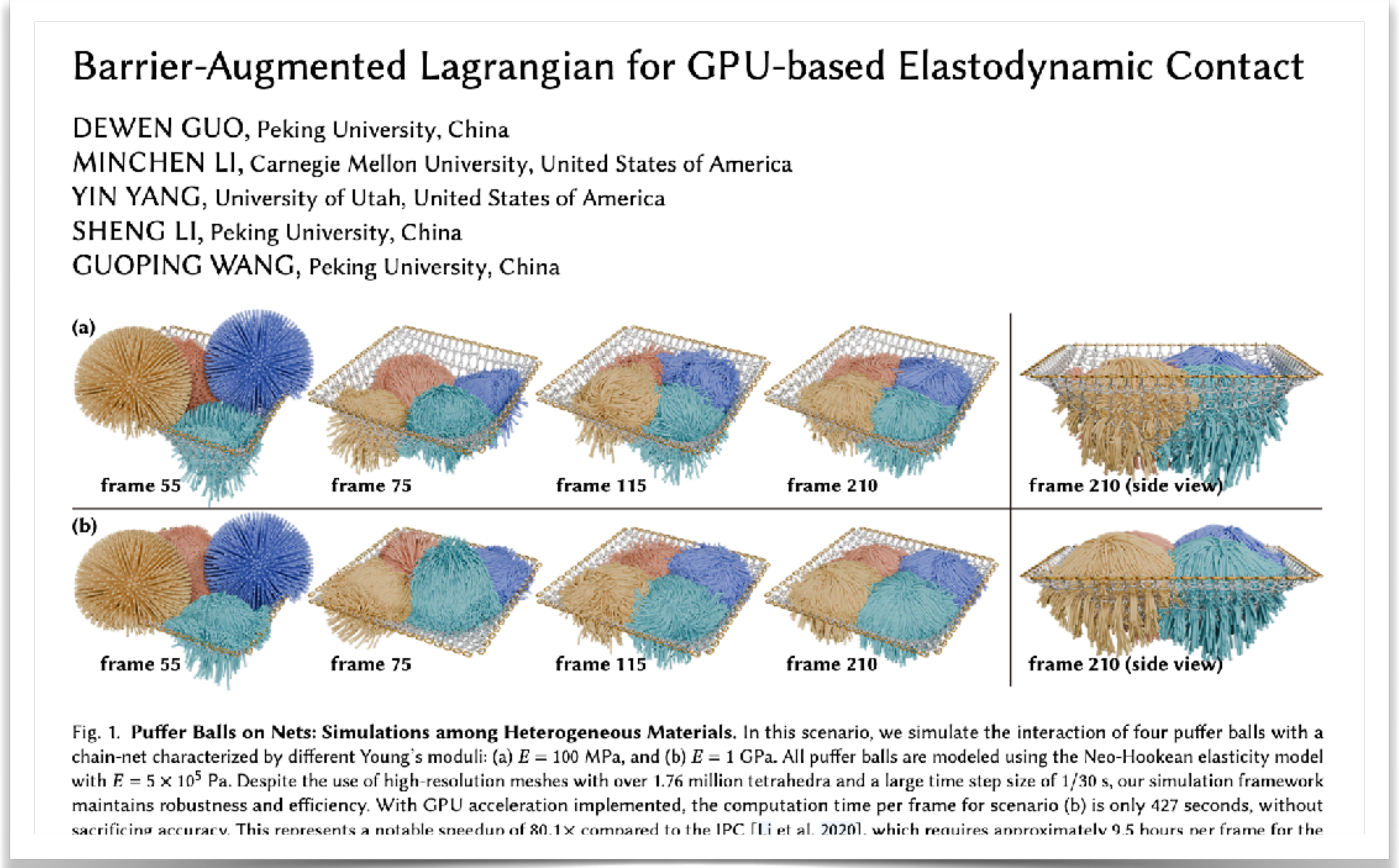
Tangjie Lv, Fuxi AI Lab, NetEase Inc, Hangzhou, Zhejiang, China, hzlvtangjie@corp.netease.com

Figure 1: Example simulation results involving complex self-collision scenarios.



Barrier-Augmented Lagrangian for GPU-based Elastodynamic Contact

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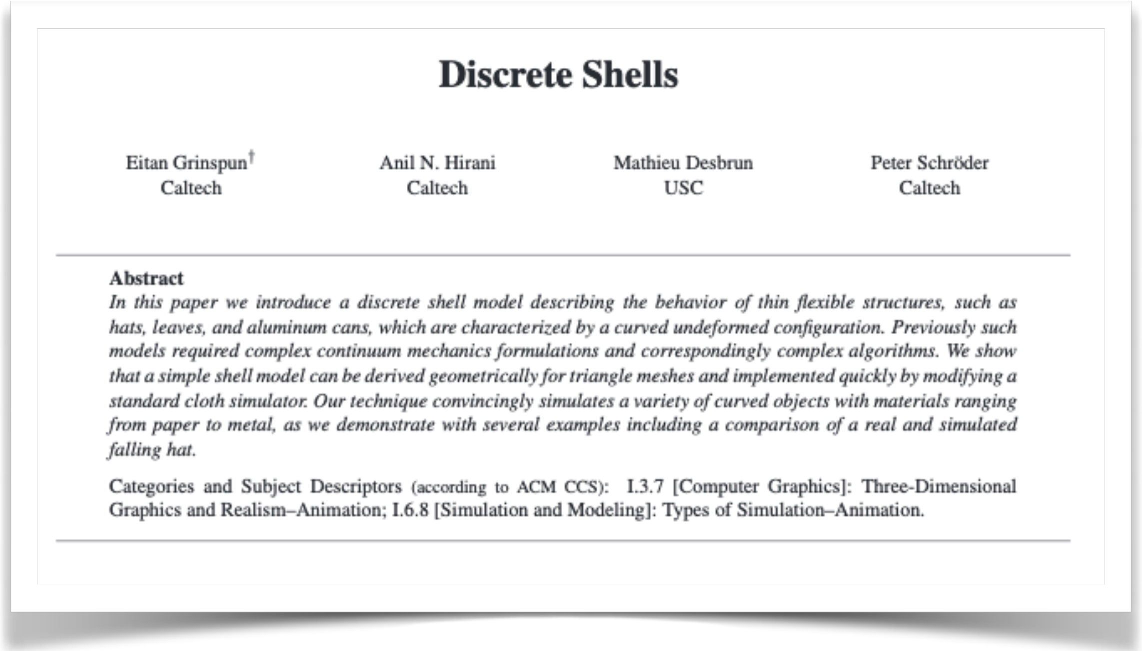
MINCHEN LI, Carnegie Mellon University, United States of America
YIN YANG, University of Utah, United States of America
SHENG LI, Peking University, China
GUOPING WANG, Peking University, China

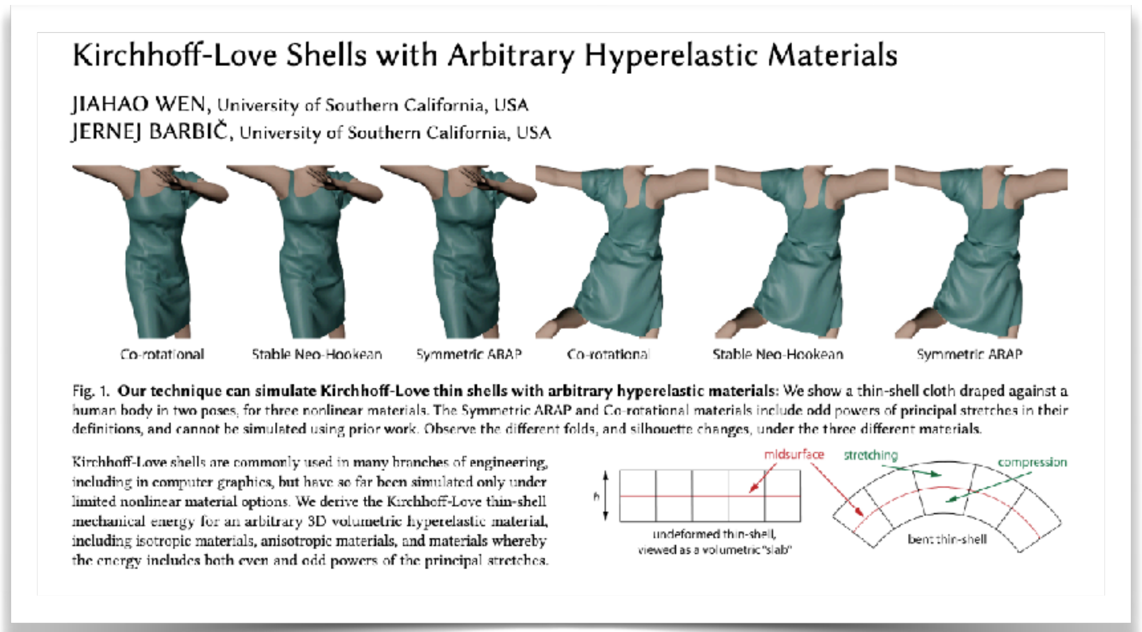
Fig. 1. **Puffer Balls on Nets: Simulations among Heterogeneous Materials.** In this scenario, we simulate the interaction of four puffer balls with a chain-net characterized by different Young's moduli: (a) $E = 100$ MPa, and (b) $E = 1$ GPa. All puffer balls are modeled using the Neo-Hookean elasticity model with $E = 5 \times 10^5$ Pa. Despite the use of high-resolution meshes with over 1.76 million tetrahedra and a large time step size of 1/30 s, our simulation framework maintains robustness and efficiency. With GPU acceleration implemented, the computation time per frame for scenario (b) is only 427 seconds, without

Cloth, Shells & Rods

Shell models (examples…)

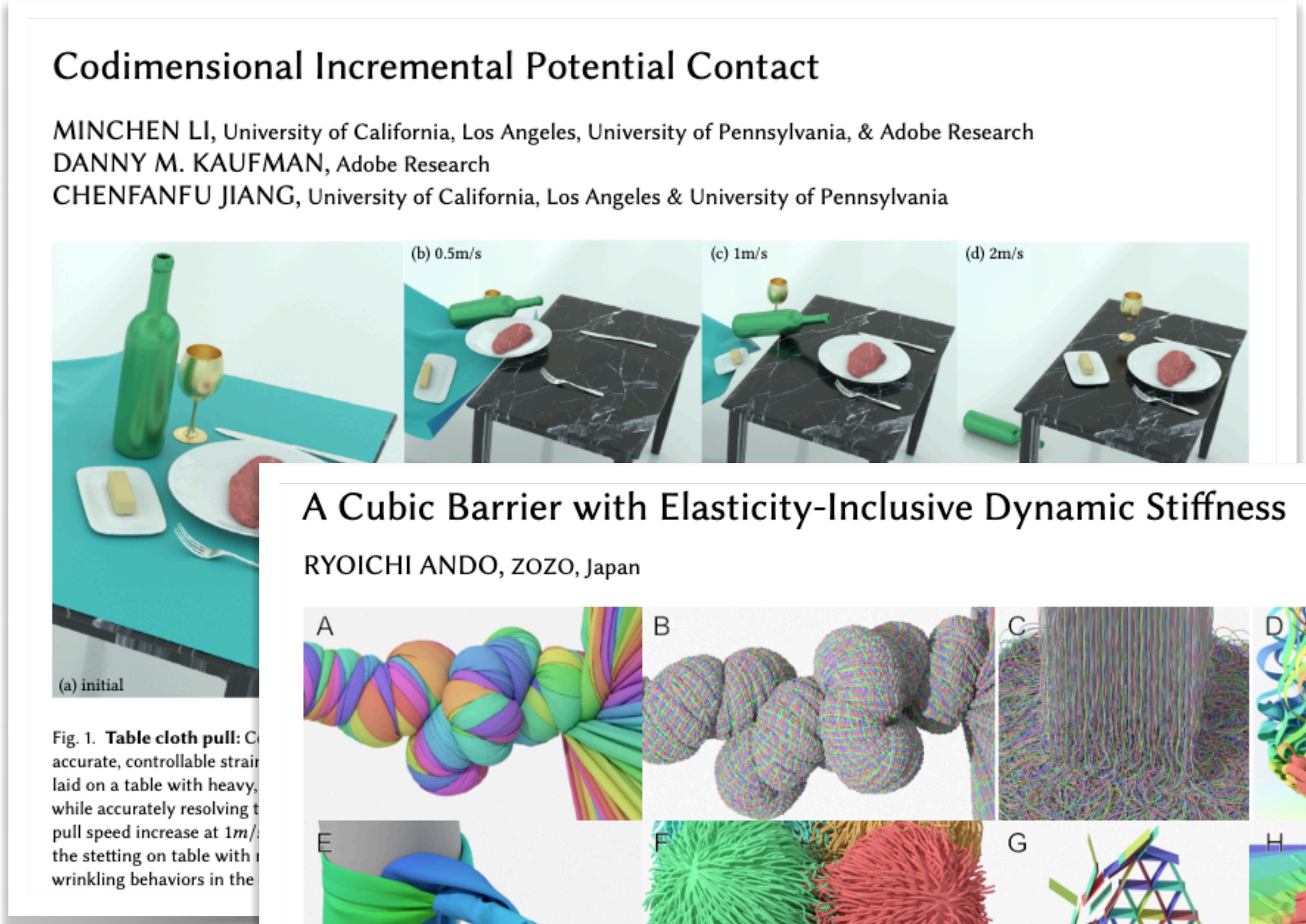


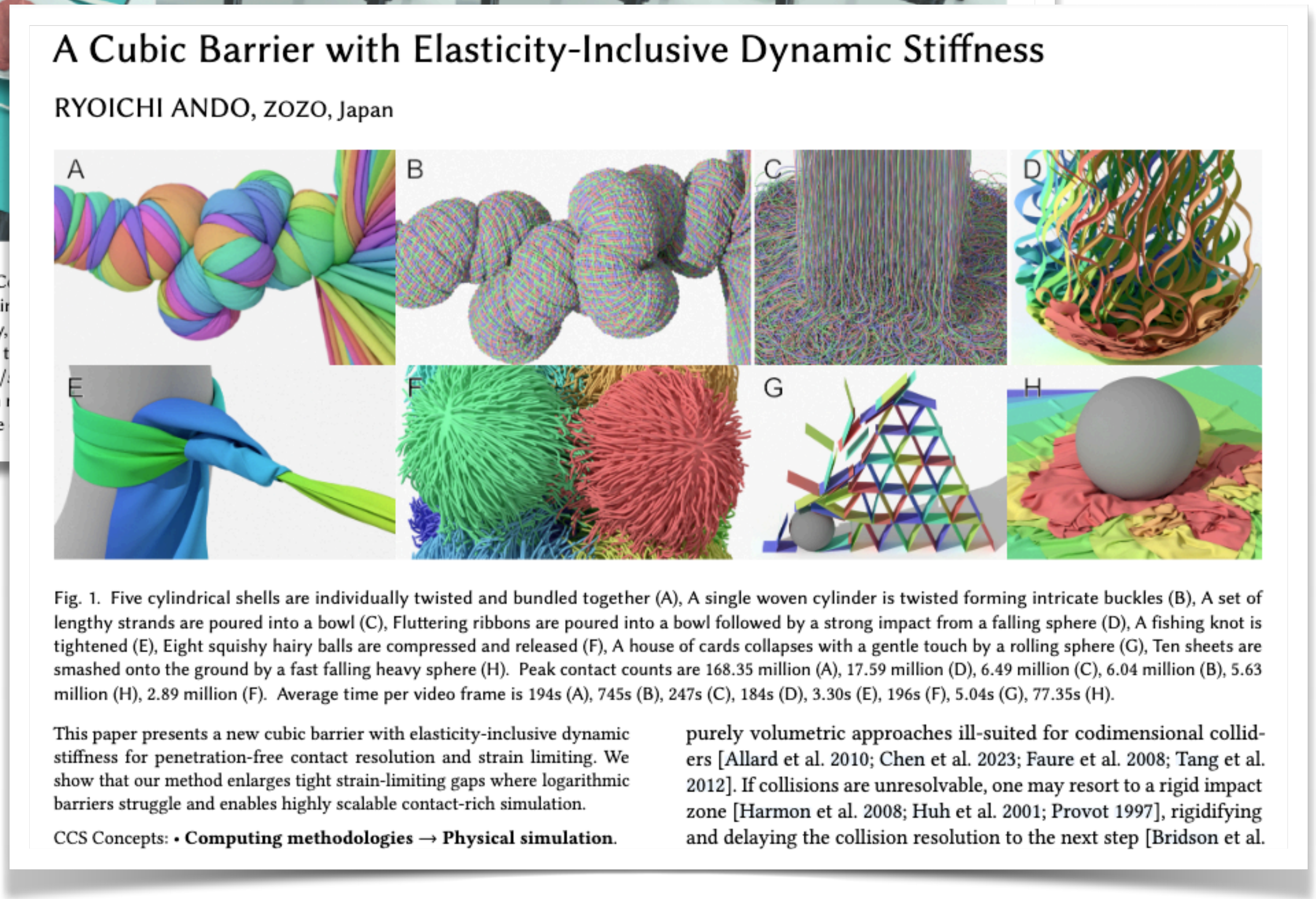
Geometrically motivated



Continuous models

Intersection-free contact





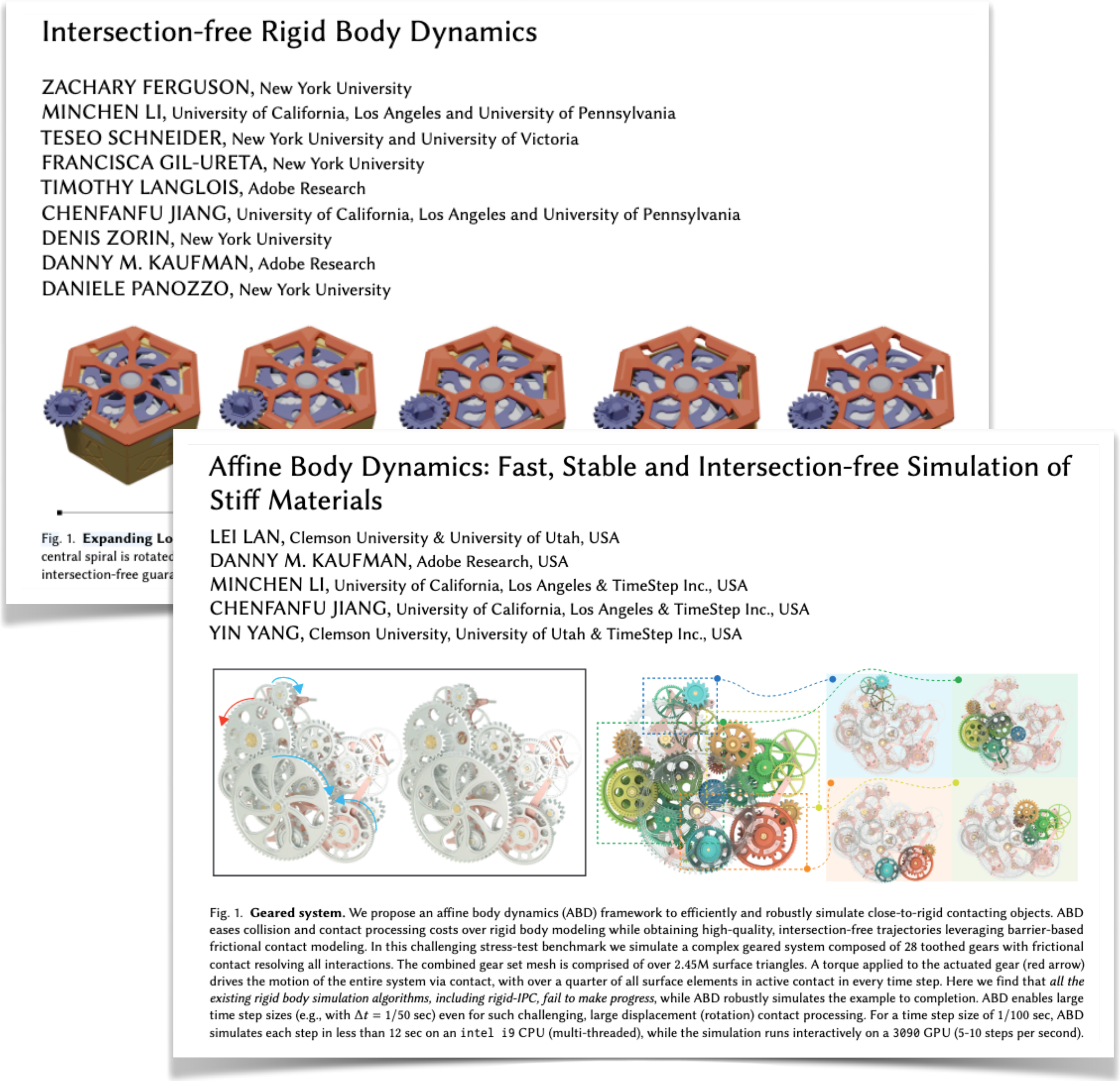
Rigid Bodies & Affine Bodies

- DOF per rigid body: translation $\boldsymbol{x}_i \in \mathbb{R}^3$ and rotation vector $\boldsymbol{\theta}_i \in \mathbb{R}^3$
- Inertial term for rotation DOF:

$$E_R(\mathbf{R}(\boldsymbol{\theta})) = \sum_{i}^{N_{\mathrm{rb}}} \left(\frac{1}{2} \mathrm{tr}(\mathbf{R}_i \mathbf{J}_i \mathbf{R}_i^T) - \mathrm{tr}(\mathbf{R}_i \mathbf{J}_i \tilde{\mathbf{R}}_i^T) \right)$$

with $\tilde{\mathbf{R}}_i = \mathbf{R}_i^{\mathrm{prev}} + h \dot{\mathbf{R}}_i^{\mathrm{prev}} + h^2 \tau_i \mathbf{J}_i^{-1}$

- Non-linear mapping from DOF to vertices
  - → Small time steps or "curved CCD" for intersection-free contact
- Alternative: **Affine Bodies**
  - → Body embedded in a single tetrahedron
  - → Use any stiff material model ($E > 10^8 \mathrm{Pa}$)


Intersection-free Rigid Body Dynamics

ZACHARY FERGUSON, New York University
MINCHEN LI, University of California, Los Angeles and University of Pennsylvania
TESEO SCHNEIDER, New York University and University of Victoria
FRANCISCA GIL-URETA, New York University
TIMOTHY LANGLOIS, Adobe Research
CHENFANFU JIANG, University of California, Los Angeles and University of Pennsylvania
DENIS ZORIN, New York University
DANNY M. KAUFMAN, Adobe Research
DANIELE PANOZZO, New York University

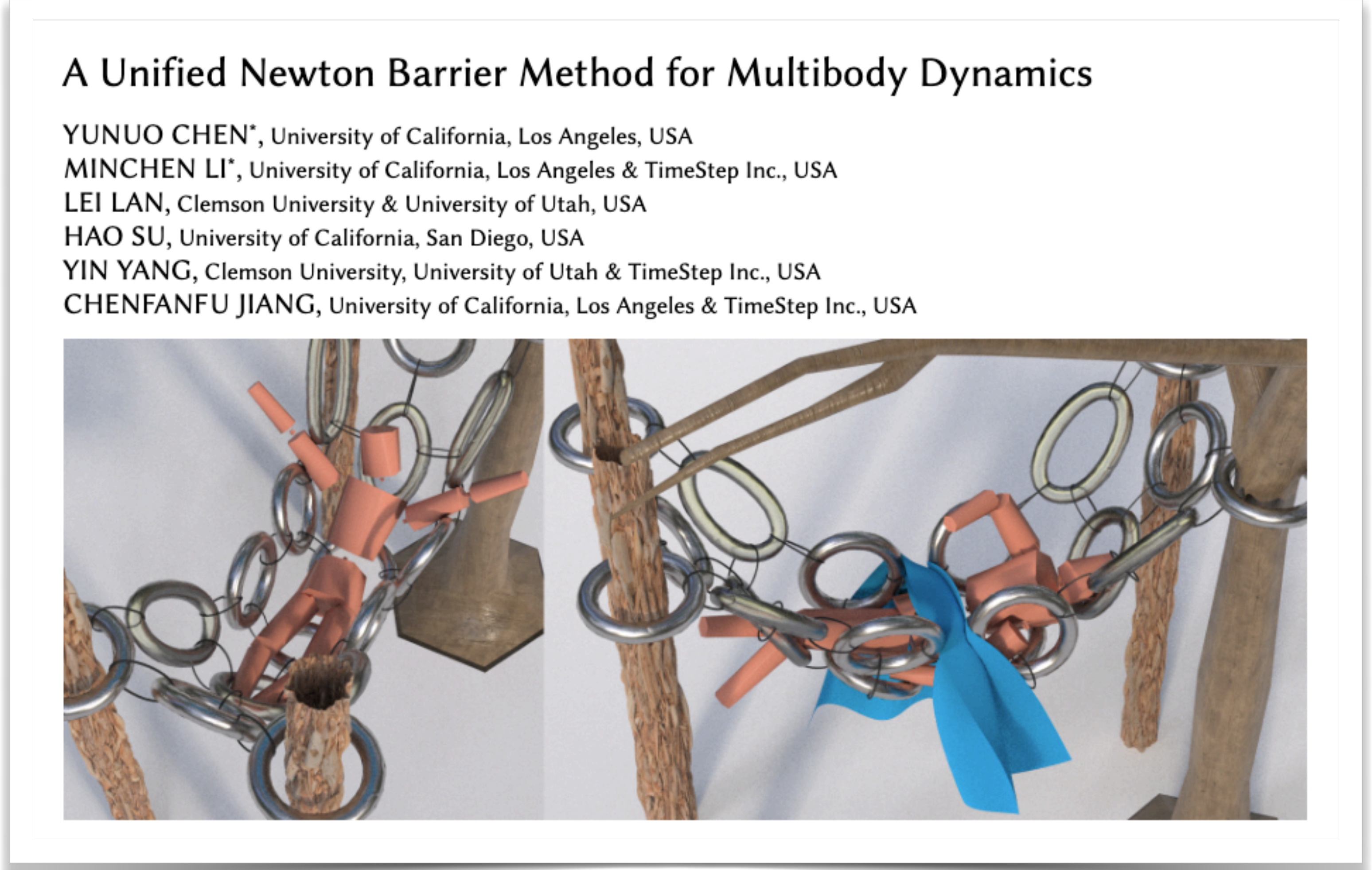
Affine Body Dynamics: Fast, Stable and Intersection-free Simulation of Stiff Materials

LEI LAN, Clemson University & University of Utah, USA
DANNY M. KAUFMAN, Adobe Research, USA
MINCHEN LI, University of California, Los Angeles & TimeStep Inc., USA
CHENFANFU JIANG, University of California, Los Angeles & TimeStep Inc., USA
YIN YANG, Clemson University, University of Utah & TimeStep Inc., USA

Fig. 1. **Geared system.** We propose an affine body dynamics (ABD) framework to efficiently and robustly simulate close-to-rigid contacting objects. ABD eases collision and contact processing costs over rigid body modeling while obtaining high-quality, intersection-free trajectories leveraging barrier-based frictional contact modeling. In this challenging stress-test benchmark we simulate a complex geared system composed of 28 toothed gears with frictional contact resolving all interactions. The combined gear set mesh is comprised of over 2.45M surface triangles. A torque applied to the actuated gear (red arrow) drives the motion of the entire system via contact, with over a quarter of all surface elements in active contact in every time step. Here we find that *all the existing rigid body simulation algorithms, including rigid-IPC, fail to make progress*, while ABD robustly simulates the example to completion. ABD enables large time step sizes (e.g., with $\Delta t = 1/50$ sec) even for such challenging, large displacement (rotation) contact processing. For a time step size of 1/100 sec, ABD simulates each step in less than 12 sec on an intel i9 CPU (multi-threaded), while the simulation runs interactively on a 3090 GPU (5-10 steps per second).

Multi-Body Systems

- Coupling deformable & rigid bodies with joints and motors
- Reformulate constraints using penalties and barriers
  - → Equality constraints: $E_{\mathrm{Eq}}(\boldsymbol{x}) = \frac{1}{2}k(c(\boldsymbol{x}))^2$
  - → Inequality constraints: $E_{\mathrm{Ineq}}(\boldsymbol{x}) = \kappa b(c(\boldsymbol{x}))$


A Unified Newton Barrier Method for Multibody Dynamics

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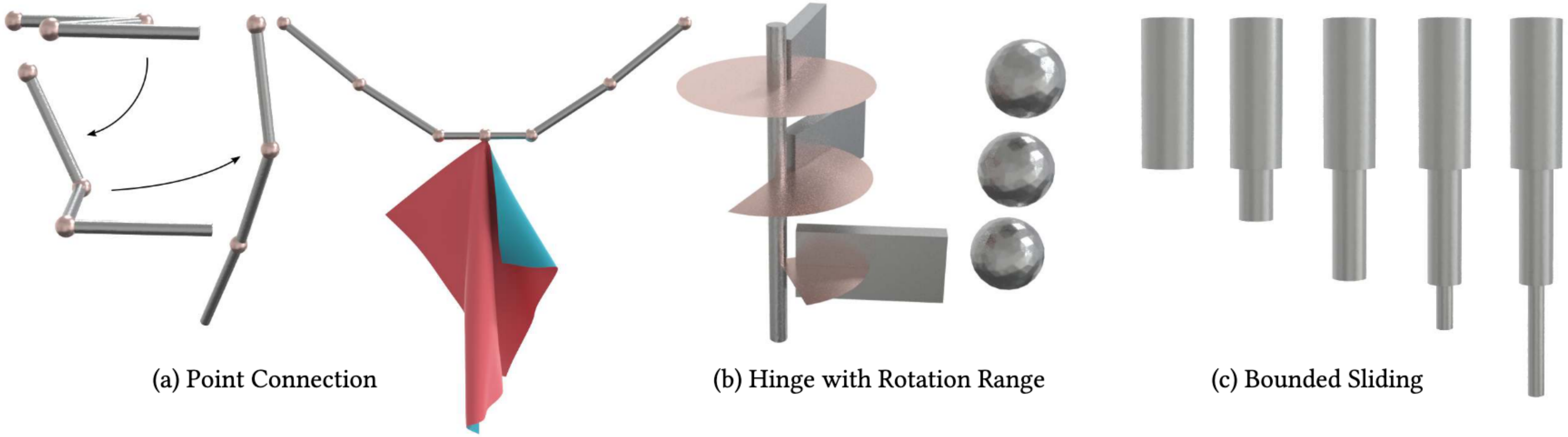
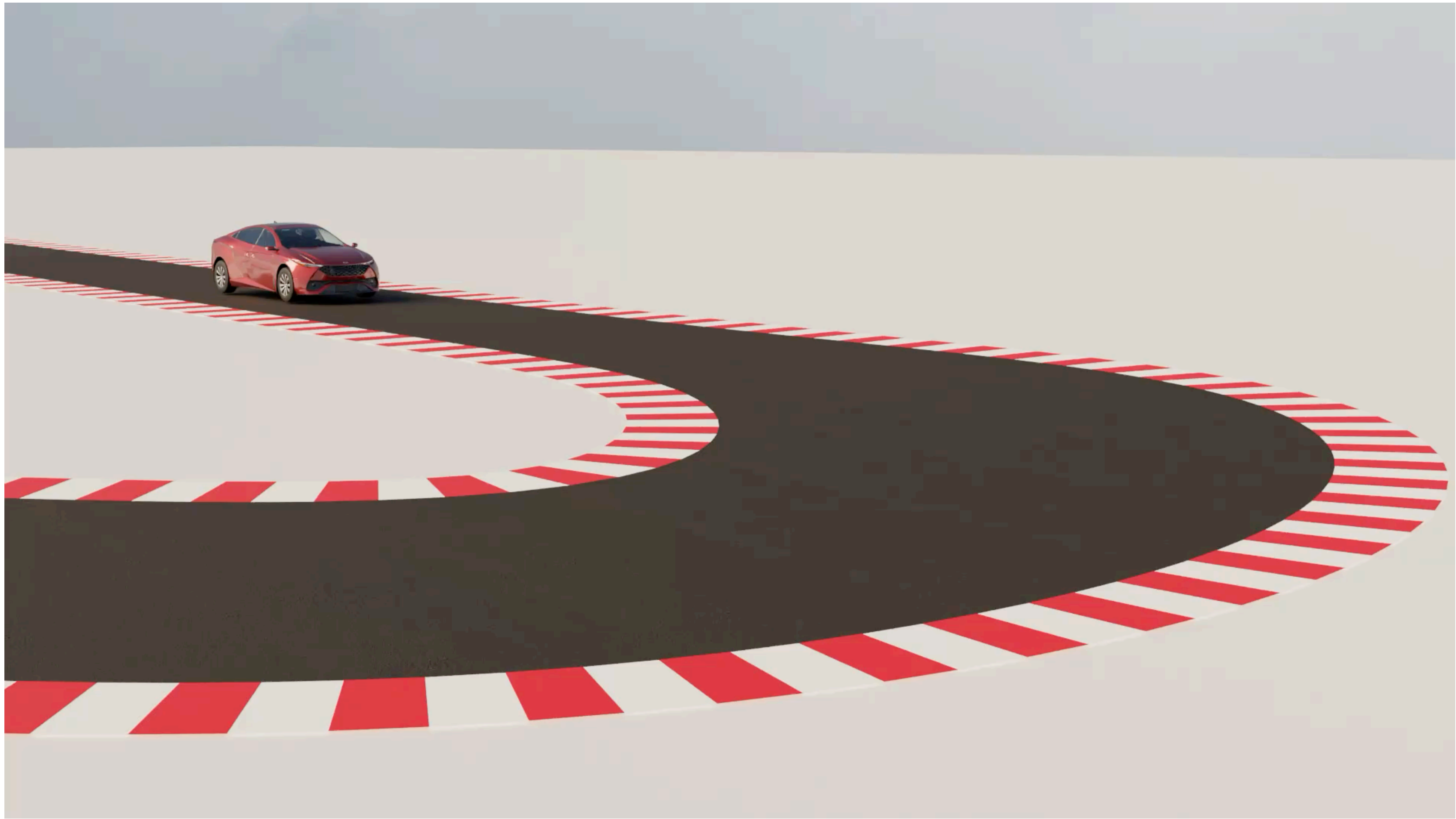
MINCHEN LI*, University of California, Los Angeles & TimeStep Inc., USA
LEI LAN, Clemson University & University of Utah, USA
HAO SU, University of California, San Diego, USA
YIN YANG, Clemson University, University of Utah & TimeStep Inc., USA
CHENFANFU JIANG, University of California, Los Angeles & TimeStep Inc., USA


(a) Point Connection (b) Hinge with Rotation Range (c) Bounded Sliding

Multi-Body Systems

Strongly coupled systems:

- Rigid bodies for car body and rims
- Tires as deformable solids
- Dampened springs and slider joints for suspension
- Connected with axial joints
- Rotation limits for steering



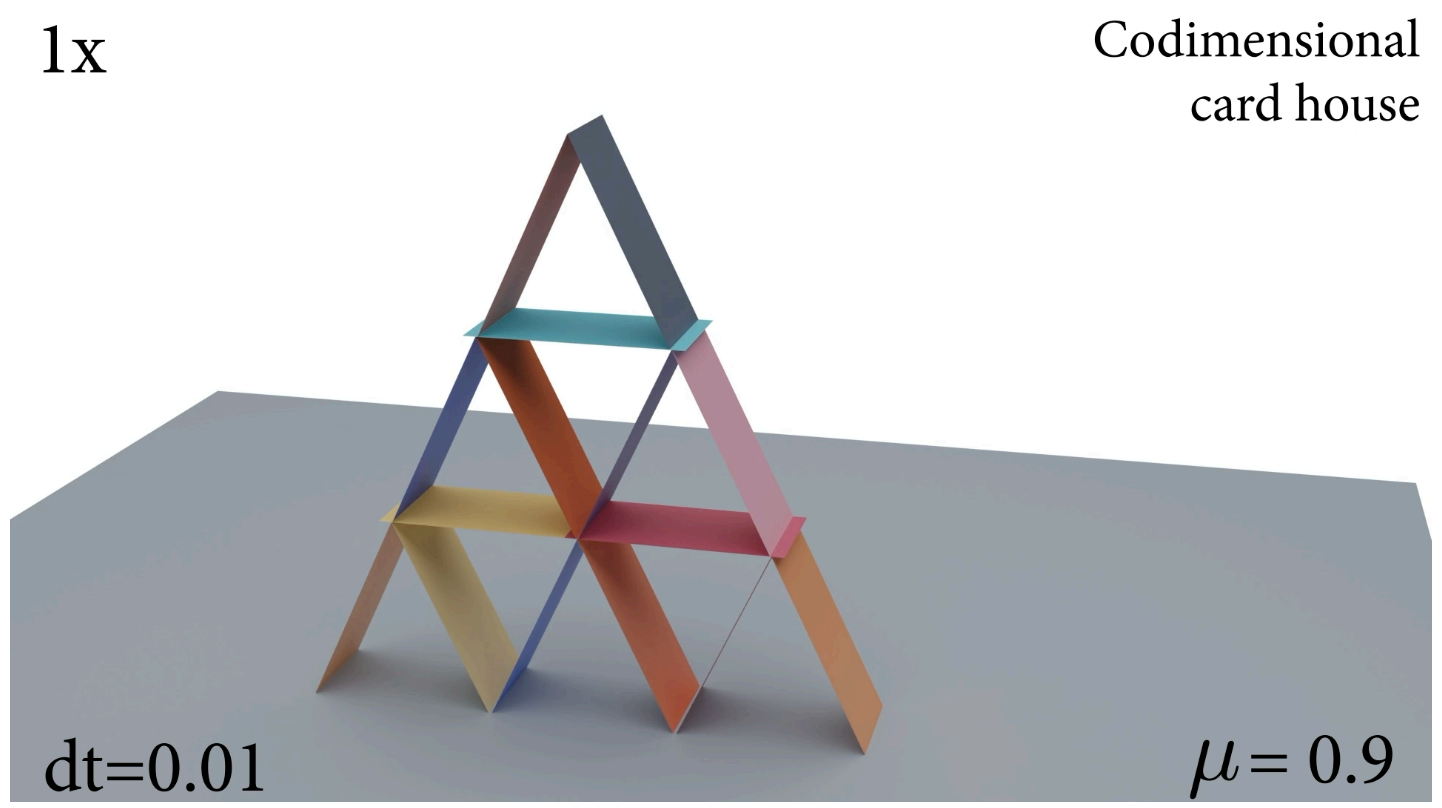
Dissipative Forces

- We need scalar potentials $\phi_i(\boldsymbol{x})$ such that $\boldsymbol{f}_i(\boldsymbol{x}) = -\nabla\phi_i(\boldsymbol{x})$

  → Possible for conservative and path-independent forces

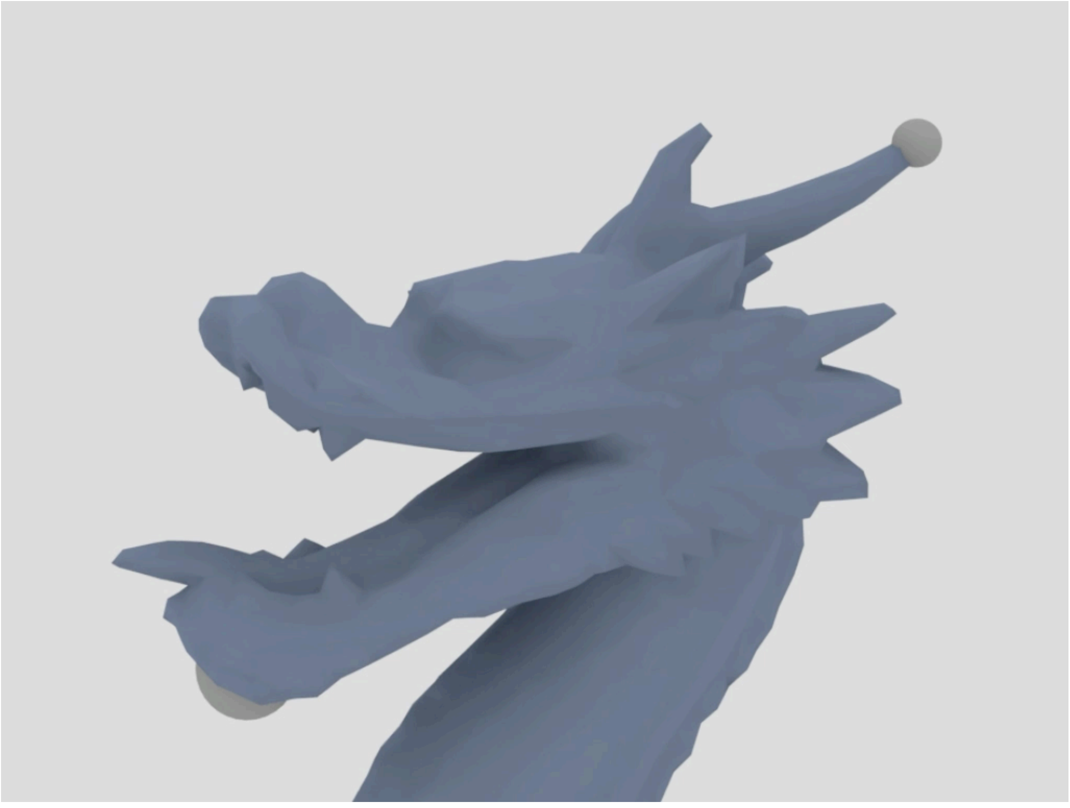
- Adaptations required for dissipative/non-conservative forces:

Friction



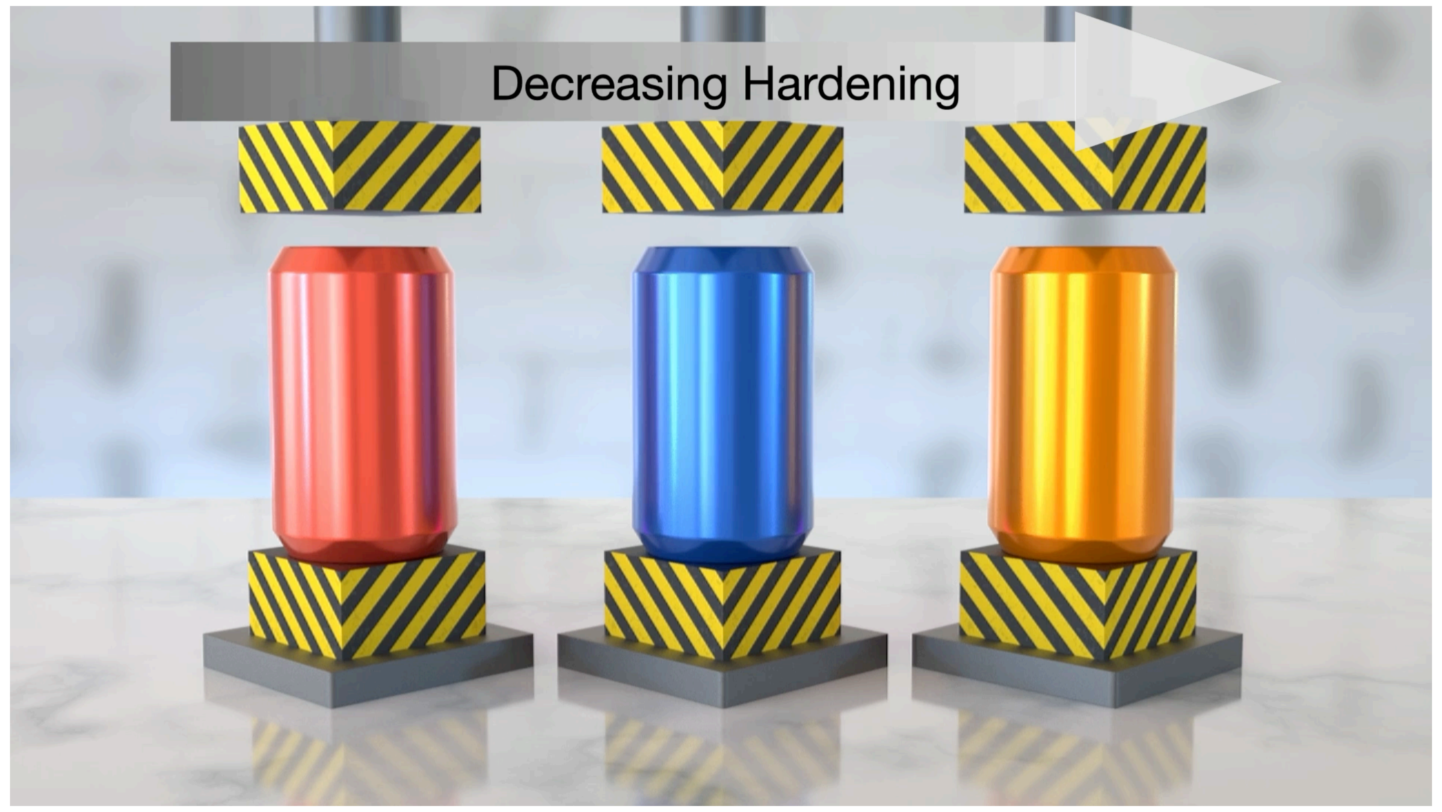
Intersection-free Rigid Body Dynamics [Ferguson et al. 2021]

Damping



Accurate Dissipative Forces in Optimization Integrators [Brown et al. 2018]

Plasticity



Energetically Consistent Inelasticity for Optimization Time Integration [Li et al. 2020]

Coupling Magnetic Effects

Magnetic Rigid Bodies

Strongly Coupled Simulation of Magnetic Rigid Bodies [Westhofen et al. 2024]

$$E_{\mathrm{magn}} = -\sum_i V_i \mathbf{M}_i \cdot \mathbf{B}_{\mathrm{ext},i}$$

Magnetoelastic Thin Shells

Simulation and Optimization of Magnetoelastic Thin Shells [Chen et al. 2022]

$$E_{\mathrm{magn}} = -\sum_i h \mathbf{F}_i \mathbf{M}_i \cdot \mathbf{B}_{\mathrm{ext},i}$$

FEM & SPH Coupling: Contact Proxy Splitting Method

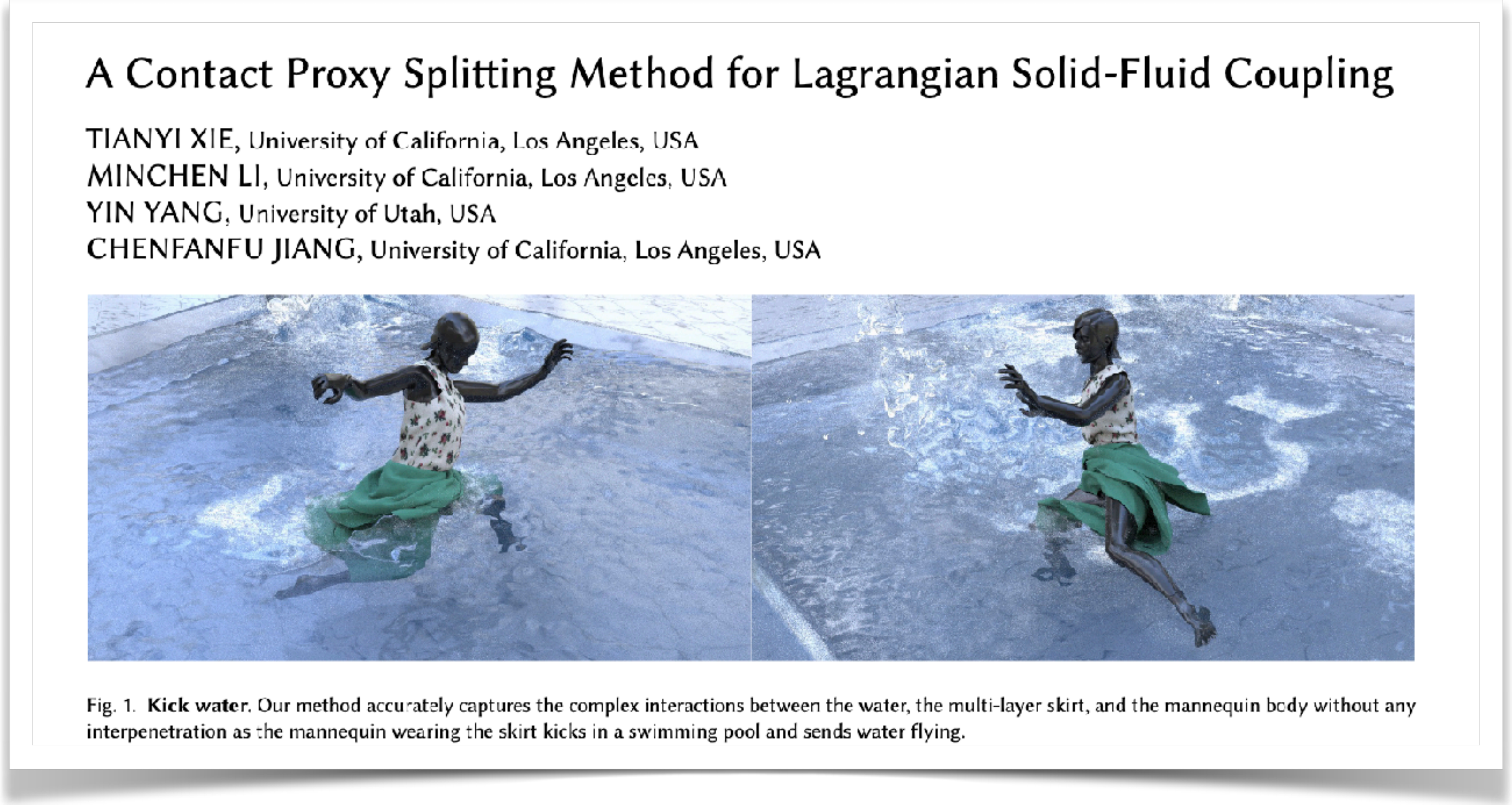
- Fully Lagrangian formulation
- Models Weakly Compressible SPH using quadratic potentials
    - e.g. incompressibility potential: $E_I = \sum_i \frac{\kappa_I}{2} V_0 \left(\frac{\rho_0}{\rho_i} - 1 \right)^2$
- Contact model: barriers on particle ↔ mesh distances

Problems of fully implicit formulation:

- "Connectivity" of SPH particles (neighbors of neighbors)
- Nonlinearity of barriers and deformables

Splitting approach:

1. Solve **fluid** phase with **linearized contact** forces
2. Solve **deformables** with **full contact** potential & **fluid proxy**



A Contact Proxy Splitting Method for Lagrangian Solid-Fluid Coupling

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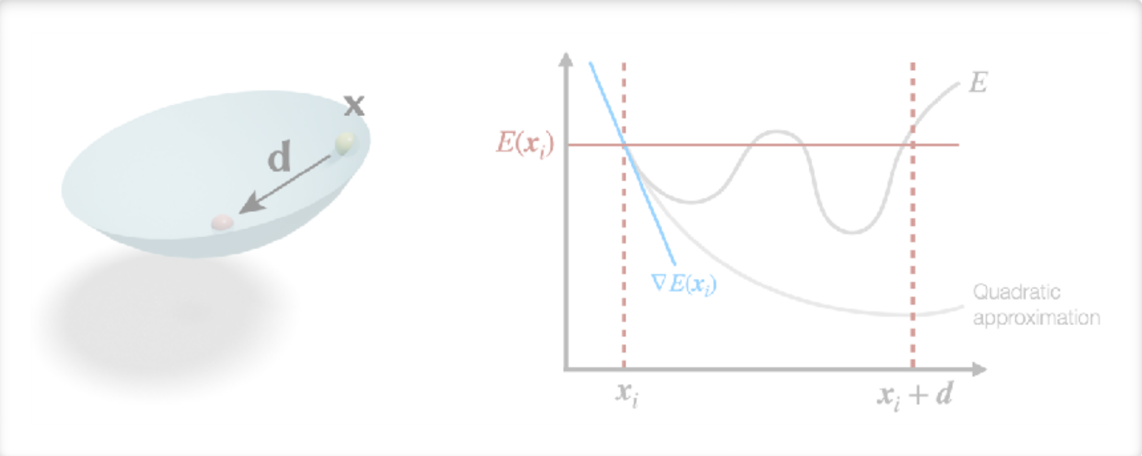
MINCHEN LI, University of California, Los Angeles, USA
YIN YANG, University of Utah, USA
CHENFANFU JIANG, University of California, Los Angeles, USA

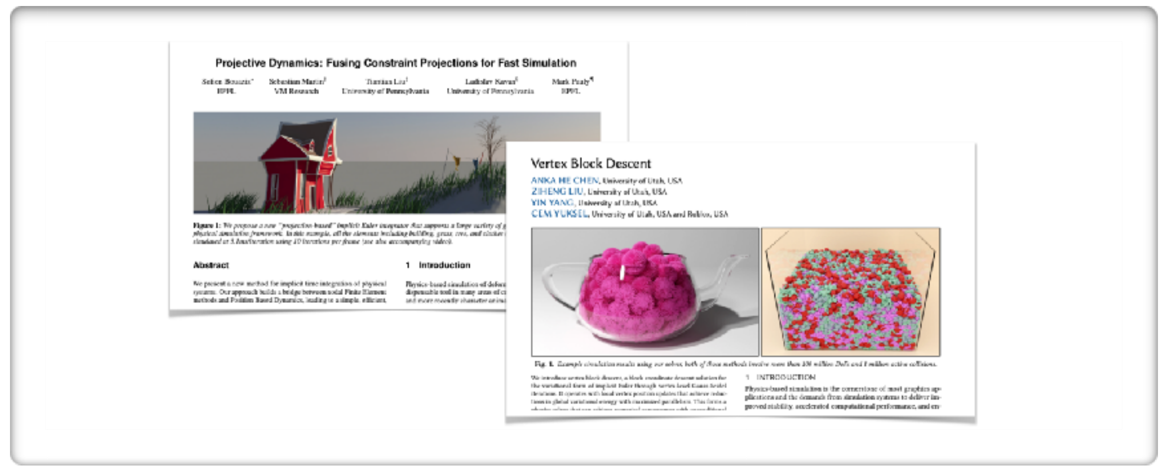
Fig. 1. **Kick water.** Our method accurately captures the complex interactions between the water, the multi-layer skirt, and the mannequin body without any interpenetration as the mannequin wearing the skirt kicks in a swimming pool and sends water flying.

Outline

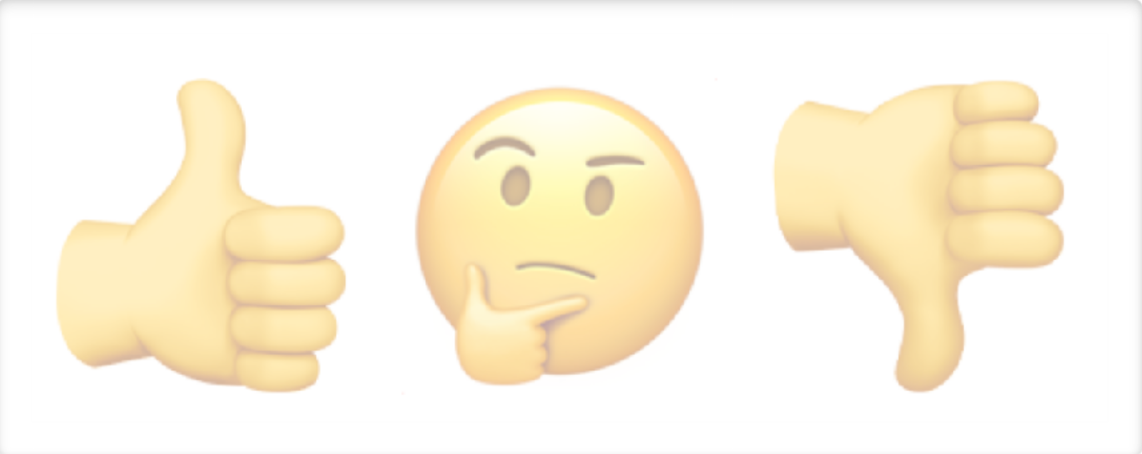


1. Mathematical foundations

2. Physical models and coupling



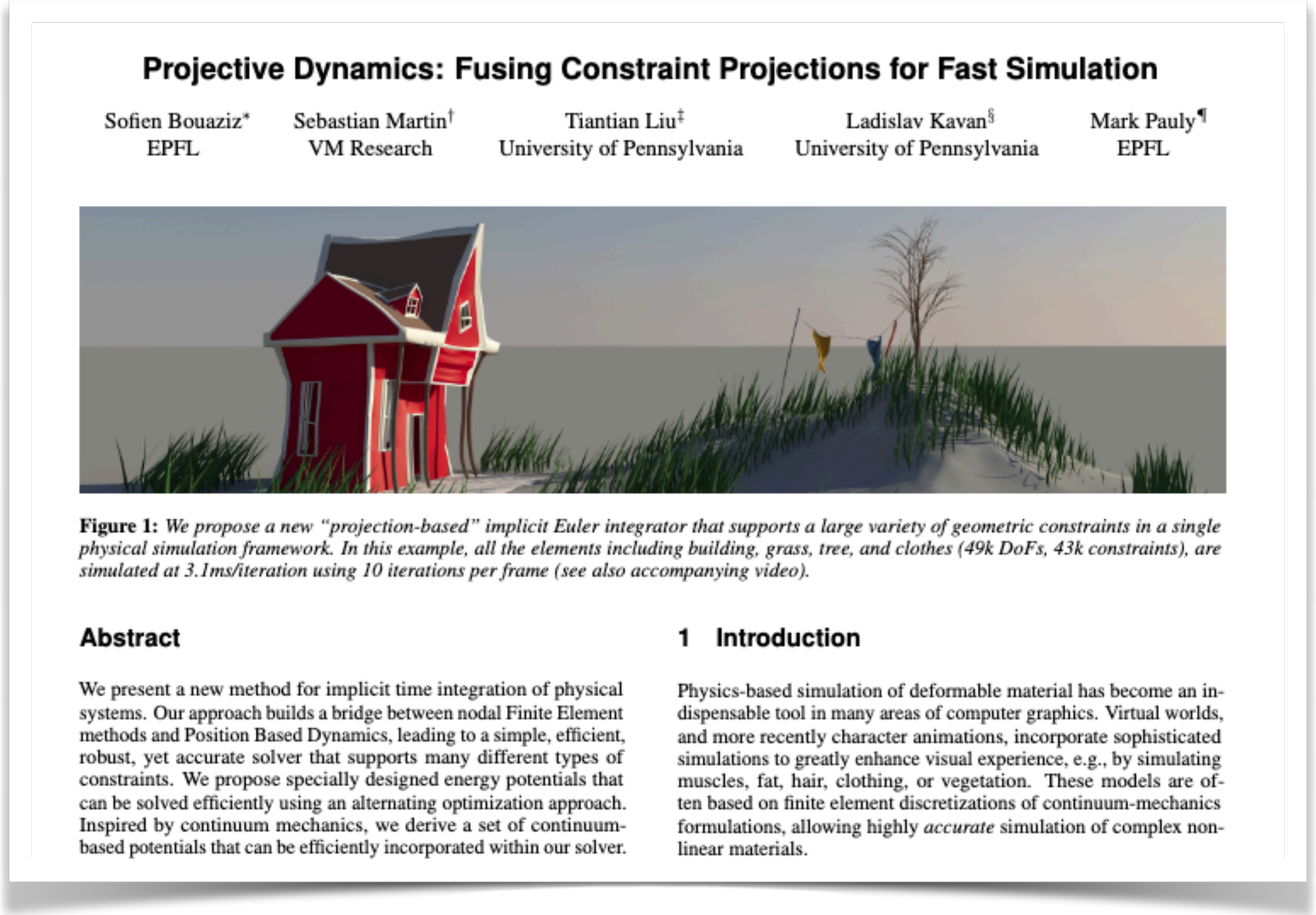
3. Related methods (VBD, PD)



4. Summary: Models & Properties

Projective Dynamics

- Introduces local/global split
- **Local:** Non-linear **constraint projections**
  - → **Per constraint,** find closest “target positions” $\boldsymbol{p}_i$ that satisfy $\boldsymbol{C}_i(\boldsymbol{p}_i) = 0$
- **Global:** for fixed $\boldsymbol{p}_i$, minimize BE **incremental potential**



Projective Dynamics: Fusing Constraint Projections for Fast Simulation

Sofien Bouaziz* EPFL — Sebastian Martin† VM Research — Tiantian Liu‡ University of Pennsylvania — Ladislav Kavan§ University of Pennsylvania — Mark Pauly¶ EPFL

Figure 1: *We propose a new “projection-based” implicit Euler integrator that supports a large variety of geometric constraints in a single physical simulation framework. In this example, all the elements including building, grass, tree, and clothes (49k DoFs, 43k constraints), are simulated at 3.1ms/iteration using 10 iterations per frame (see also accompanying video).*

Abstract

We present a new method for implicit time integration of physical systems. Our approach builds a bridge between nodal Finite Element methods and Position Based Dynamics, leading to a simple, efficient, robust, yet accurate solver that supports many different types of constraints. We propose specially designed energy potentials that can be solved efficiently using an alternating optimization approach. Inspired by continuum mechanics, we derive a set of continuum-based potentials that can be efficiently incorporated within our solver.

1 Introduction

Physics-based simulation of deformable material has become an indispensable tool in many areas of computer graphics. Virtual worlds, and more recently character animations, incorporate sophisticated simulations to greatly enhance visual experience, e.g., by simulating muscles, fat, hair, clothing, or vegetation. These models are often based on finite element discretizations of continuum-mechanics formulations, allowing highly *accurate* simulation of complex nonlinear materials.

$$E_{\mathrm{PD}}(\boldsymbol{x},\boldsymbol{p}) = \frac{1}{2\Delta t^2}(\boldsymbol{x}-\tilde{\boldsymbol{x}})^{\top}\mathbf{M}(\boldsymbol{x}-\tilde{\boldsymbol{x}}) + \sum_i d_i(\boldsymbol{x},\boldsymbol{p}_i)$$

$\boxed{E_{\mathrm{PD}} \text{ has Constant Hessian} \rightarrow \text{Pre-Factorize}}$

→ Where $d_i(\boldsymbol{x},\boldsymbol{p}_i)$ are **quadratic** functions, pulling the DOF $\boldsymbol{x}$ towards $\boldsymbol{p}_i$

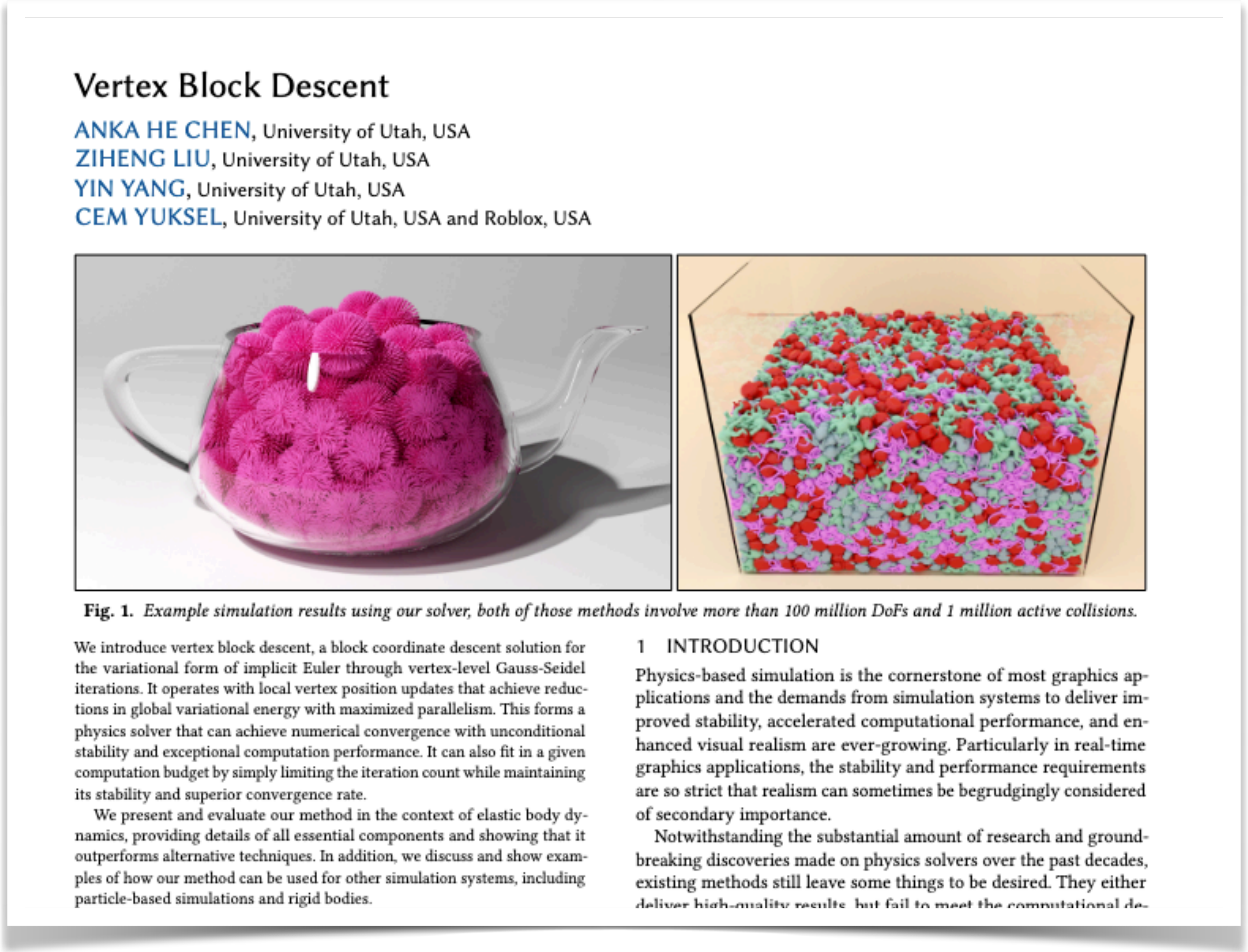
Vertex Block Descent

- Uses a block coordinate descent approach
- Solves **local** (per-vertex) energies:

$$E_i(\boldsymbol{x}) = \frac{m_i}{2\Delta t^2}\|\boldsymbol{x}_i - \tilde{\boldsymbol{x}}_i\|^2 + \boxed{\sum_{j \in \mathcal{V}_i} \phi_j(\boldsymbol{x})}$$

Energies affecting vertex i

- Analytic inversion of local **3x3 Hessians**
- Designed for efficient implementation on GPU
- Alternative to XPBD, but is a **primal** method
- Reduces global energy but does not necessarily converge to its minimum



Vertex Block Descent

ANKA HE CHEN, University of Utah, USA
ZIHENG LIU, University of Utah, USA
YIN YANG, University of Utah, USA
CEM YUKSEL, University of Utah, USA and Roblox, USA

Fig. 1. *Example simulation results using our solver, both of those methods involve more than 100 million DoFs and 1 million active collisions.*

We introduce vertex block descent, a block coordinate descent solution for the variational form of implicit Euler through vertex-level Gauss-Seidel iterations. It operates with local vertex position updates that achieve reductions in global variational energy with maximized parallelism. This forms a physics solver that can achieve numerical convergence with unconditional stability and exceptional computation performance. It can also fit in a given computation budget by simply limiting the iteration count while maintaining its stability and superior convergence rate.

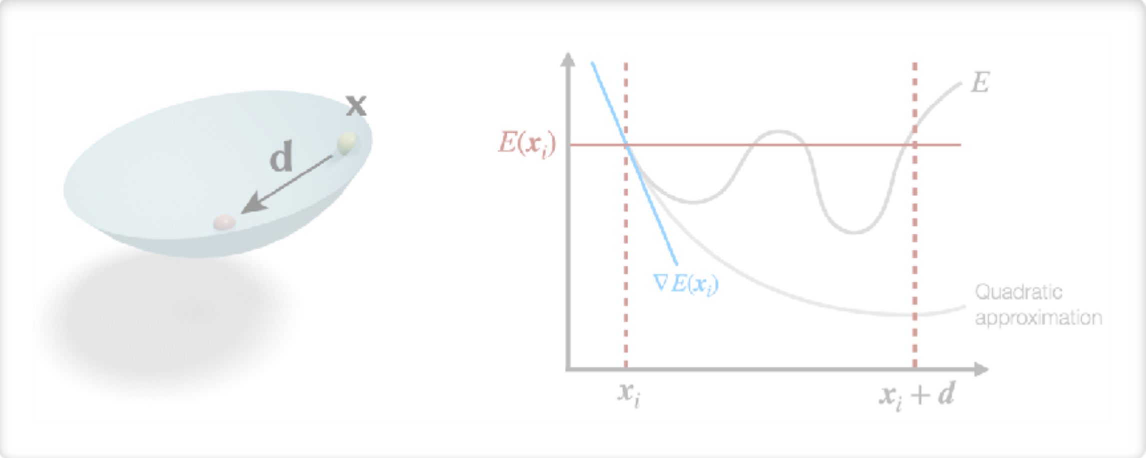
We present and evaluate our method in the context of elastic body dynamics, providing details of all essential components and showing that it outperforms alternative techniques. In addition, we discuss and show examples of how our method can be used for other simulation systems, including particle-based simulations and rigid bodies.

1 INTRODUCTION

Physics-based simulation is the cornerstone of most graphics applications and the demands from simulation systems to deliver improved stability, accelerated computational performance, and enhanced visual realism are ever-growing. Particularly in real-time graphics applications, the stability and performance requirements are so strict that realism can sometimes be begrudgingly considered of secondary importance.

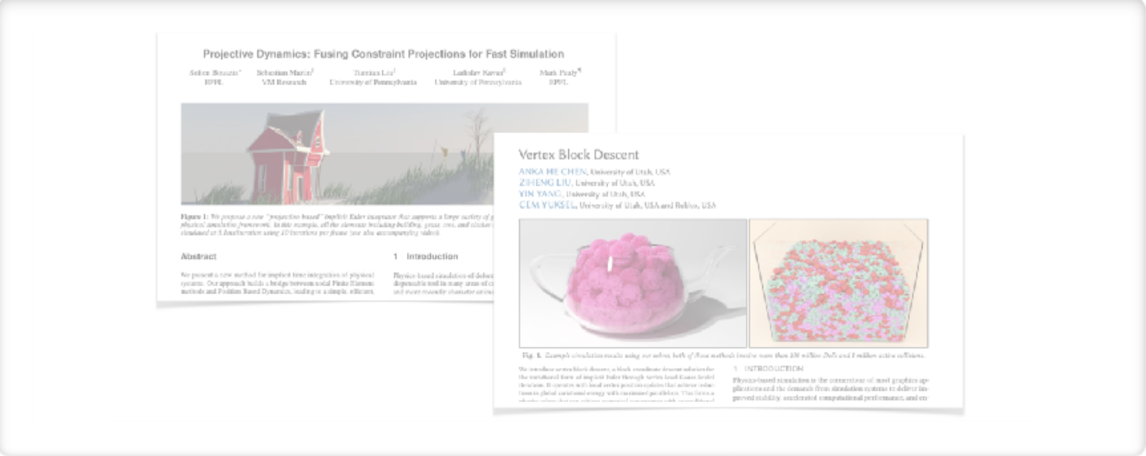
Notwithstanding the substantial amount of research and groundbreaking discoveries made on physics solvers over the past decades, existing methods still leave some things to be desired. They either deliver high-quality results, but fail to meet the computational de-

Outline

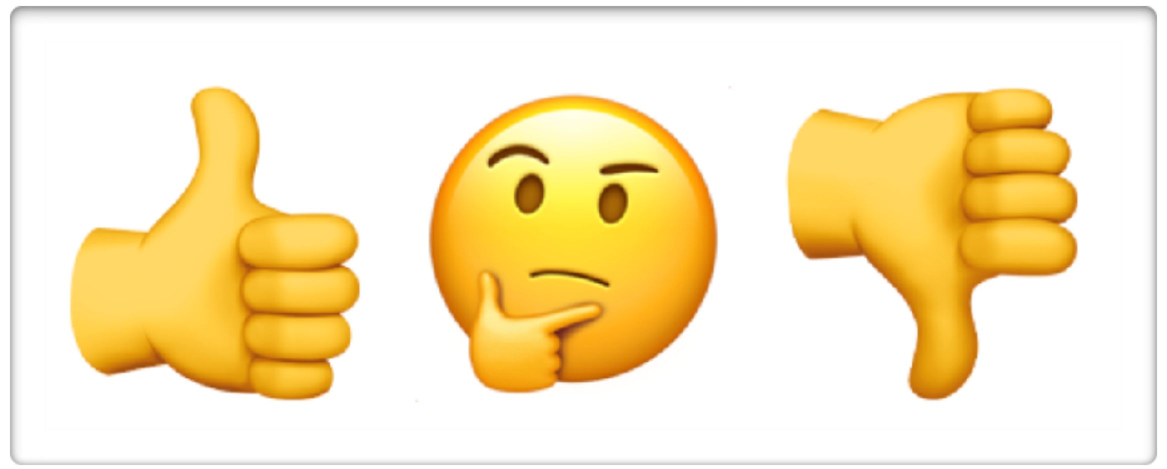


1. Mathematical foundations

2. Physical models and coupling



3. Related methods (VBD, PD)



4. Summary: Models & Properties

Summary: Energy-based Models

- 🌟 Deformable solids, cloth, shells and rods
  - 👍 Geometrically motivated & continuum-based models
  - 👍 Material models: isotropic, anisotropic, example-based, data-driven…
- 👍 Dynamic frictional contact & intersection-free simulations
- 👍 Rigid bodies & multi-body systems
- 👍 Other physical phenomena (e.g. magnetics, heating, wetting…)
- 🤔 "Accurate" friction and certain plasticity models
  - → Typically needs approximations (e.g. lagging)
- 🤔 Fluids & granular media
  - → Typically needs operator splitting, semi-implicit approaches or constrained optimization

Summary: Properties of Energy-Based Simulation

👍 **Simple:** in terms of formulation and numerical approach

👍 **Robust convergence:** due to line search & second-order derivatives

🤔 **Primal methods:** Good for high mass ratios, very large stiffness ratios numerically challenging

👎 **Limited to potentials:** no arbitrary dissipative and non-conservative forces

- 👉 Solution: Adapted and approximate models (for friction, damping and plasticity)

$$f(\boldsymbol{x}) = -\nabla_{\boldsymbol{x}} \phi(\boldsymbol{x})$$

or

$$f_D(\boldsymbol{v}) = -\nabla_{\boldsymbol{v}} \phi_D(\boldsymbol{v})$$

👎 **High computational cost:** problematic for interactive applications

- 👉 Solution: Related methods for interactive applications, e.g.:
  - VBD: Vertex Block Descent [Chen et al. 2024]
  - PD: Projective Dynamics [Bouaziz et al. 2014]

Part II
Energy-based multiphysics modeling

Daniel Holz[1,2], Stefan Rhys Jeske[3], **Fabian Löschner**[3], Jan Bender[3], Yin Yang[4], Sheldon Andrews[2]

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